Stabilizing Queues in Large-scale Networks

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Abstract— **We present a novel constrained flow control scheme for a class of large-scale networks, modelled by interconnected network resources with capacity and buffer size limitations. We first propose a decentralized sliding mode controller [7] to achieve asymptotic regulation of each individual network node in the presence of uncertain inter-node traffic, while network delays are omitted. We then incorporate network delays associated with inter-node transmissions and study both the delay-independent and the delay-dependent control designs, to investigate the delay robustness of the proposed flow control scheme. It is shown that the when network delays are small, asymptotic stability can be preserved under the decentralized, constrained control law, using the idea of delay-dependent design. A delay-independent regulation scheme is also presented to counteract arbitrary network delays.**

Index Terms— **Bottleneck queue regulation, decentralized, delays, congestion control, nonlinear, capacity constraints.**

I. INTRODUCTION

Controlling traffic in communication networks has been a popular topic for more than a decade, see [9][6][12] and [11] for existing results. Since it is in general difficult to accurately model large-scale networks with heterogeneous end-hosts, interests have been expressed to study a relatively simple network model. Nonetheless, the most important performance related features–such as bottleneck resource dynamics, physical constraints and network delays–shall be accounted for.

In this paper, we focus on the model of a class of large-scale networks which captures the above-described features of intermediate network resources. The network core is modelled as meshed interconnections of network nodes. We study the problem of decentralized output-port queue regulation at each node due to its important impact on the quality of service for "premium traffic", including audio, video, and teleconferencing transmissions on the Internet. Our analysis and design are based on a nonlinear network model and on applying Lyapunov's direct method [7], as opposed to the frequency domain approaches appeared in previous linearization-based studies for Internet congestion control [3][8][6][13]. The contributions of our paper are two-fold: 1) Our decentralized control laws achieve robust asymptotic regulation for each node against unknown inter-node traffic and against transmission delays. 2) The capacity and buffer size constraints set performance restrictions to networks and deserve to be carefully addressed. Our work provides a case study on applying nonlinear control techniques for large-scale networks with saturation constraints and delays.

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II. PROBLEM FORMULATION AND PRELIMINARIES

We discuss here the queue length regulation problem at the output-port of a network node. By "node" we mean, for example, an intermediate router or switch, or a wireless mobile unit. The following model uses the conservation law to establish the dynamic equation for queue length at a network node.

$$\dot{x}(t) = -\mu(x(t)) + \lambda(t).$$

Ensemble average queue length x is taken as the state variable, its size is limited by $x \in [0, x_{buffer}]$ with x_{buffer} being the maximum buffer size. The first function $\mu(x)$ denotes the average departure rate of the node. The second term $\lambda(t)$ stands for the average incoming traffic rate. It is unknown *a priori*, but satisfies a certain known bound.

The choice of $\mu(x) = \frac{x}{1+x}C$, where C stands for the to-be-assigned capacity for the node, is first introduced in [1] and then studied in the literature for the purpose of network performance evaluation and control under non-stationary conditions [12][10][4]. It is based upon very general network assumptions and can be used to model a wide range of communication networks [12].

In the light of the above discussion of a single-node system, our model for a large-scale network follows naturally as an interconnected system composed of n nodes, each described by subsystem S_i $(1 \leq i \leq n)$:

$$\dot{x}_i = -\frac{x_i}{1+x_i} \cdot C_i + \lambda_i\left(t, x_1(t-\tau_{i1}), \cdots, x_n(t-\tau_{in})\right), \quad (1)$$

$$x_i \in [0, x^{[i]}_{buffer}], \quad (2)$$

$$C_i(t) \in \left[0, C^{[i]}_{server}\right]. \quad (3)$$

x_i, the average queue length of the i-th node, is the state variable and is subject to the constraint (2), where $x^{[i]}_{buffer}$ is the buffer size limitation of the node. C_i, the to-be-assigned capacity is chosen as the control input of the i-th subsystem, it is subject to the saturation constraint (3) where $C^{[i]}_{server}$ denotes the maximum available capacity. λ_i denotes unknown inter-node traffic (affected by queue lengths of individual resources) and other modelling uncertainties. It is treated as to-be-rejected disturbances.

We suppose the transmission delay τ_{ij} between node i and node j satisfies $0 \leq \tau_{ij} \leq \tau_{max},\ \forall i, j = 1, ..., n$. The initial conditions of the above delay differential equations are

$$x_i(\theta) = \phi_i(\theta), \forall \theta \in [-\tau_{max}, 0],$$

where $\phi_i(\cdot)$ is a nonnegative continuous function. Note that when $\tau_{max} = 0$, the above model describes a delay-free ideal large-scale network.

In the sequel, we denote $\bar{x}_i(t) := x_i(t) - x_i^{ref}$ the regulation error of node i between the instantaneous queue length and the reference value, denoted by $x_i^{ref} \in (0, x_{buffer}^{[i]}]$. Similarly, we introduce $\bar{x}_i(t-\tau_{ij}) := x_i(t-\tau_{ij}) - x_i^{ref}$. It is assumed that the reference value x_i^{ref} for each individual node is given.

The objective of our design is to find the control $C_i(x_i)$ that accomplishes the regulation task. Namely, we want to achieve that $\bar{x}_i(t) \to 0$ $(x_i(t) \to x_i^{ref})$ as $t \to \infty$ for all initial queue lengths, by using a feedback control law $C_i(x_i)$, under the constraint $0 \leq C_i \leq C_{server}^{[i]}$, while uncertain but bounded time-varying disturbances $\lambda_i(t)$ are present. This requirement is known as "asymptotic regulation" in the control community. The asymptotic bottleneck queue regulation is pertinent for meeting the stringent service requirement of audio, video, and teleconferencing Internet transmissions. The service constraints on queuing delay, jitter, and packet loss ratio can be translated into an appropriately chosen reference value [10]. The control scheme is required to be decentralized (Namely each controller only accesses its local queue length information.) to ease the implementation complexity in an entirely distributed environment.

We introduce the key assumption of this paper followed by discussions on its physical implications.

Assumption 1: $\forall i = 1, ..., n$, $\lambda_i \geq 0$ satisfies a saturation-based Lipschitz-like condition, i.e.,

$$|\lambda_i(t, y_1, ..., y_n) - \lambda_i(t, y_1', ..., y_n')| \leq \sum_{j \neq i}^{n} \gamma_{ij} sat\left\{\sigma_{ij}(|\tilde{y}_j|)\right\}, \quad (4)$$

$$\lambda_i(t, 0, ..., 0) = 0, \ \forall t \geq 0 \quad (5)$$

where $\tilde{y}_j = y_j - y_j'$, $\sigma_{ij}(\cdot)$ is a continuously differentiable function that satisfies $\sigma_{ij}(0) = 0, \forall i, j \in \{1, ..., n\}$. Furthermore, for each i and for any fixed $t_0 \geq 0$, the following inequality holds where $X = [x_1, ..., x_n]^T$:

$$x_i^{ref} < \int_{t_0}^{\infty} \lambda_i(t, X) dt \leq \infty. \quad (6)$$

Assume for every i,

$$\sum_{j=1, j \neq i}^{n} \gamma_{ij} < \frac{x_i^{ref}}{1 + x_i^{ref}} C_{server}^{[i]}. \quad (7)$$

Remark 1: The queue length of each node affects its transmission rate, and hence, affects the inter-node traffic, reflected by the function $\sigma_{ij}(\cdot)$. "sat" is adopted here to highlight the impact of capacity constraints. It is commonly used to represent saturation constraints and is defined as $\text{sat}(y) = \min\{|y|, 1\} sgn(y)$. The constant coefficient $\gamma_{ij} \geq 0$ denotes the impact of other physical factors on the disturbance traffic from node j to node i, including locations of nodes, distances among them, and connectivity etc. (6) is known as a "persistent excitation"(PE) [7] requirement in control theory. It implies that the network is sufficiently utilized in the long run. Given the high traffic volume in modern communication networks, we believe the above PE requirement is not too restrictive.

III. Robust control design for large-scale networks

Asymptotic regulation for the large-scale network is approached via the saturated decentralized control laws

$$C_i(x_i) = \max\left\{0, C_{server}^{[i]} \cdot sat\left[\alpha_i \bar{x}_i + \beta_i sgn(\bar{x}_i)\right]\right\}, \quad (8)$$

where the parameters $\alpha_i > 0$ and $\beta_i > 0$ are to be determined.

We first study the control design for delay-free networks ($\tau_{max} = 0$), then propose a delay-independent control design to handle network delays ($0 < \tau_{ij} \leq \tau_{max}, 1 \leq i, j \leq n$). The idea of delay-independent control design is to force the network to be stable for arbitrarily large delays. We will comment on the conservativeness of the delay-independent design to motivate a different delay-robust control scheme– the delay-dependent design.

A. Control Design for Delay-free Networks

Assuming that $\tau_{max} = 0$, we now analyze the closed-loop system performance under the control law (8). We will show that with appropriately chosen control parameters α_i and β_i, asymptotic regulation of the large-scale network is achieved for all possible initial conditions.

Before we present the main result on the regulation of large-scale networks, we first introduce some facts that are useful for analyzing the closed-loop system performance.

Lemma 1: Consider the closed-loop large-scale network system composed of (1) and (8). Suppose Assumption 1 holds and in the control law (8),

$$\alpha_i > \frac{1 - \beta_i}{x_{buffer}^{[i]} - x_i^{ref}}, \ 0 \leq \beta_i < 1. \quad (9)$$

For all $x_i(t_0) \in [0, x_{buffer}^{[i]}], t_0 \geq 0$, $x_i(t)$ satisfies (2), $\forall t \geq t_0$. Furthermore, there exists some finite $T \geq t_0$ such that $\forall t \geq T$,

$$X(t) \in \Omega \doteq \left\{X \in \Re^n | x_i^{ref} \leq x_i \leq x_i^*, \forall i = 1, ..., n\right\}, \quad (10)$$

where $x_i^* = \frac{1-\beta_i}{\alpha_i} + x_i^{ref} > x_i^{ref}$.

Proof: It can be directly checked that $x_i^{ref} < x_i^* < x_{buffer}^{[i]}$. We first analyze the case when $x_i(t_0) < x_i^{ref}$. When $x_i(t) < x_i^{ref}$, it holds $\dot{x}_i(t) = \lambda_i(t) \geq 0$. Thus for any $x_i(t_0) < x_i^{ref}$, under the "PE" condition in Assumption 1, there exists finite $t' > t_0$ such that $x_i^{ref} \leq x_i(t') \leq x_i^*$.

We then consider the case when $x_i(t_0) > x_i^*$. For all $x_i(t) \in (x_i^*, x_{buffer}^{[i]}]$, $\alpha_i \bar{x}_i(t) + \beta_i sgn(\bar{x}_i(t)) > 1$, thus $C_i(x_i(t)) = C_{server}^{[i]}$. It holds, by applying Assumption 1,

$$0 \leq \lambda_i(t, x_1, ..., x_n) \leq \sum_{j \neq i}^{n} \gamma_{ij} sat\left\{\sigma_{ij}(x_j)\right\} < \frac{x_i^{ref}}{1 + x_i^{ref}} C_{server}^{[i]}.$$

$$\begin{aligned} \dot{x}_i(t) &= -\frac{x_i(t)}{1 + x_i(t)} C_{server}^{[i]} + \lambda_i(t) \\ &< -\frac{x_i^*}{1 + x_i^*} C_{server}^{[i]} + \frac{x_i^{ref}}{1 + x_i^{ref}} C_{server}^{[i]} < 0. \end{aligned}$$

It follows that for any $x_i(t_0) > x_i^*$, there exists some finite $t'' > t_0$ such that $x_i^{ref} \leq x_i(t'') \leq x_i^*$. The above analysis leads to that for all $x_i(t_0) \in [0, x_{buffer}^{[i]}]$, there must exists finite $t^* \geq t_0$ such that $x_i^{ref} \leq x_i(t^*) \leq x_i^*$. We now prove by contradiction that $x_i^{ref} \leq x(t) \leq x_i^*, \forall t \geq t^*$. In fact, suppose $x_i(t) > x_i^*$ or $x_i(t) < x_{ref}^{[i]}$ for some $t > t^*$, there

must exists some $t_1 \geq t^*$ such that either one of the following two statements is true:
S1. $x_i(t_1) = x_i^*$ and $x_i(t_1+\tau) > x_i^*$ for some $\tau > 0$, or
S2. $x_i(t_1) = x_i^{ref}$ and $x_i(t_1+\tau) < x_i^{ref}$ for some $\tau > 0$.

When $x_i = x_i^*$, $\alpha_i \bar{x}_i + \beta_i sgn(\bar{x}_i) = 1$, thus $C_i(x^*) = C_{server}^{[i]}$. It holds:

$$\begin{aligned} \dot{x}_i &= -\frac{x_i^*}{1+x_i^*} C_{server}^{[i]} + \lambda_i(t) \\ &< -\frac{x_i^{ref}}{1+x_i^{ref}} C_{server}^{[i]} + \sum_{j \neq i}^{n} \gamma_{ij} \leq 0. \end{aligned}$$

This contradicts S1. When $x_i = x_i^{ref}$, $C_i(x_i^{ref}) = 0$. Thus $\dot{x}_i = \lambda_i \geq 0$, which contradicts S2. We can conclude that $x_i^{ref} \leq x_i(t) \leq x_i^*, \forall t \geq t^*$. In other words, $\left\{x_i | x_i^{ref} \leq x_i \leq x_i^*\right\}$ is an attractive forward invariant set. The above analysis reveals that $x_i(t)$ satisfies (2), $\forall t \geq t_0$.

We can apply such analysis to every node i to show that every $x_i(t)$ satisfies (2) and there exists some finite t_i^* for every node $i \in \{1, ..., n\}$, such that $x_i^{ref} \leq x_i(t) \leq x_i^*, \forall t \geq t_i^*$. By setting $T \doteq \max\{t_i^* | i = 1, ..., n\}$, we have $X(t) \in \Omega, \forall t \geq T$. In other words, Ω is an attractive forward invariant set. ■

For notational conveniences, denote P the $n \times n$ matrix with elements $p_{ij}, i, j = 1, ..., n$,

$$\begin{aligned} p_{ii} &= \frac{x_i^{ref}}{1+x_i^{ref}} C_{server}^{[i]} \alpha_i,\ p_{ij} = -\frac{1}{2}\left(\bar{\gamma}_{ij} + \bar{\gamma}_{ji}\right), \quad (11) \\ \bar{\gamma}_{ij} &= \max_{0 \leq \bar{x}_j \leq \frac{1-\beta_i}{\alpha_i}} \frac{\gamma_{ij} sat\{\sigma_{ij}(\bar{x}_j)\}}{\bar{x}_j}, i, j = 1, ..., n, i \neq j. \end{aligned}$$

$\bar{\gamma}_{ij}$ is finite because of the definition of "sat" function and the property of function σ_{ij}. Let

$$\begin{aligned} \upsilon_i &:= \sum_{j \neq i}^{n} \gamma_{ij} sat\{\sigma_{ij}(x_j^{ref})\}, \\ \epsilon_i &:= \frac{\upsilon_i (1+x_i^{ref})}{C_{server}^{[i]} x_i^{ref}} < 1, i = 1, ..., n. \end{aligned}$$

Theorem 1: Suppose in (1), $\tau_{max} = 0$. Consider the closed-loop system composed of (1) and (8) where α_i, β_i satisfy (9) and α_is are chosen such that P is positive definite. Suppose that the interfering traffic λ_i satisfies Assumption (1). If $\beta_i \in [\epsilon_i, 1), \forall i \in \{1, ..., n\}$, the queue length x_i of every node converges to x_i^{ref} asymptotically for all $x_i(t_0) \in [0, x_{buffer}^{[i]}]$.

Proof: By the definition of υ_i and Assumption 1,

$$\upsilon_i \leq \sum_{j=1, j \neq i}^{n} \gamma_{ij} < \frac{x_i^{ref}}{1+x_i^{ref}} C_{server}^{[i]}, i = 1, ..., n.$$

Thus by the definition of ϵ_i, there exists β_i such that $\epsilon_i \leq \beta_i < 1$ for every i. By the definition of P, α_is can be chosen such that $P > 0$. According to Lemma 1, trajectories of the closed-loop system satisfy (2), and are ultimately confined within Ω. We now analyze the system for $t \geq T$. Combining (4) and (5), we arrive at

$$\lambda_i(t, x_1, ..., x_n) \leq \sum_{j=1, j \neq i}^{n} \gamma_{ij} sat\left\{\sigma_{ij}(\bar{x}_j)\right\} + \upsilon_i.$$

Since $x_i(t) \in [x_i^{ref}, x_i^*]$, $\alpha_i \bar{x}_i(t) + \beta_i sgn(\bar{x}_i(t)) \leq 1, \forall i = 1, ..., n$,

$$C_i(x_i(t)) = C_{server}^{[i]}\left[\alpha \bar{x}_i(t) + \beta_i sgn(\bar{x}_i(t))\right].$$

Consider the function $V = \frac{1}{2}\sum_{i=1}^{n} \bar{x}_i^2$. It holds, by differentiating V to t along the closed-loop system trajectories and using the definitions of P and $\bar{\gamma}_{ij}$,

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{n} \bar{x}_i \Big(-\frac{x_i}{1+x_i} C_i + \lambda_i\Big) \\ &\leq \sum_{i=1}^{n} \bar{x}_i \Big[-\frac{x_i^{ref}}{1+x_i^{ref}} C_{server}^{[i]}\left[\alpha_i \bar{x}_i + \beta_i sgn(\bar{x}_i)\right] + \\ &\quad \sum_{j=1, j \neq i}^{n} \gamma_{ij} sat\left\{\sigma_{ij}(\bar{x}_j)\right\} + \upsilon_i\Big] \\ &\leq -\sum_{i=1}^{n} \frac{x_i^{ref} C_{server}^{[i]}}{1+x_i^{ref}} \alpha_i \bar{x}_i^2 + \sum_{i=1}^{n} \bar{x}_i \sum_{j=1, j \neq i}^{n} \bar{\gamma}_{ij} \bar{x}_j \\ &\quad -\sum_{i=1}^{n}\left(\frac{x_i^{ref}}{1+x_i^{ref}} C_{server}^{[i]} \beta_i - \upsilon_i\right)|\bar{x}_i| \\ &\leq -\bar{X}^T P \bar{X} - \sum_{i=1}^{n}\left(\frac{x_i^{ref}}{1+x_i^{ref}} C_{server}^{[i]} \beta_i - \upsilon_i\right)|\bar{x}_i|, \end{aligned}$$

where $\bar{X} = [\bar{x}_1, ..., \bar{x}_n]^T$.

Applying the fact that $P > 0$ and $\beta \geq \epsilon_i, \forall i = 1, .., n$ leads to $\dot{V} \leq 0$. By Barbălat's Lemma [7], we conclude that $\lim_{t \to \infty} \sum_{i=1}^{n} |\bar{x}_i(t)| = 0$. ■

B. Delay-independent Design for Networks with Delays

For notational conveniences, we introduce

$$\begin{aligned} \rho_i &:= \frac{x_i^{ref}}{1+x_i^{ref}} C_{server}^{[i]} \alpha_i - \sum_{j \neq i}^{n} \frac{\bar{\gamma}_{ij} + \bar{\gamma}_{ji}}{2}, \quad (12) \\ \nu_i &:= \frac{x_i^{ref}}{1+x_i^{ref}} C_{server}^{[i]} \beta_i - \upsilon_i. \end{aligned}$$

Theorem 2: Suppose in (1), $\tau_{ij} > 0, 1 \leq i, j \leq n$. Consider the closed-loop large-scale network system under control laws $C_i(x_i)$ and Assumption 1. If α_i and β_i are chosen such that $\rho_i > 0, \nu_i \geq 0, i = 1, ..., n$, it holds that the trajectories of the large-scale network converge asymptotically to $\{X \in \Re^n | x_i = x_i^{ref}, i = 1, ..., n\}$ in the face of arbitrary delays $\tau_{ij}, i, j \in [1, n]$.

Proof: Consider the Lyapunov-Krasovskii functional candidate:

$$V = \frac{1}{2}\sum_{i=1}^{n} \bar{x}_i^2(t) + \sum_{i=1}^{n}\sum_{j \neq i}^{n} \int_{t-\tau_{ij}}^{t} \frac{\bar{\gamma}_{ij}}{2} \bar{x}_j^2(s) ds.$$

$\bar{x}_i(t) = x_i(t) - x_i^{ref}$. Similarly with Lemma 1, we can prove that there exists some finite $T \geq 0$, such that for all $t \geq T$, $x_i(t) \in [x_i^{ref}, x_i^*]$. Thus on $[T, \infty)$, $\alpha_i \bar{x}_i(t) + \beta_i sgn(\bar{x}_i(t)) \leq 1$, so

$$C_i(x_i(t)) = C_{server}^{[i]}\left[\alpha_i \bar{x}_i(t) + \beta_i sgn(\bar{x}_i(t))\right]\ \forall i = 1, ..., n.$$

By application of the above assumption to the interconnection term $\lambda_i(t, x_1(t-\tau_{i1}), ..., x_n(t-\tau_{in}))$, we can arrive at

$$\lambda_i(t, x_1(t-\tau_{ij}), x_2(t-\tau_{ij}), ..., x_n(t-\tau_{ij})) \leq \sum_{j=1, j\neq i}^{n} \gamma_{ij} sat\{\sigma_{ij}(\bar{x}_j(t-\tau_{ij}))\} + \upsilon_i.$$

We now start differentiating V along the solutions of the closed-loop system. We consider the closed-loop system on $[T+\tau_{max}, \infty)$.

$$\begin{aligned}\dot{V} =& \sum_{i=1}^{n} \bar{x}_i \left(-\frac{x_i}{1+x_i} C_i + \lambda_i\right) \\ &+ \sum_{i=1}^{n}\sum_{j\neq i}^{n} \left[\frac{\bar{\gamma}_{ij}}{2}\bar{x}_j^2(t) - \frac{\bar{\gamma}_{ij}}{2}\bar{x}_j^2(t-\tau_{ij})\right] \\ \leq& -\sum_{i=1}^{n} \frac{x_i^{ref} C_{server}^{[i]}}{1+x_i^{ref}} \alpha_i \bar{x}_i^2(t) + \sum_{i=1}^{n}\sum_{j\neq i}^{n} \bar{\gamma}_{ij}\bar{x}_i(t)\bar{x}_j(t-\tau_{ij}) \\ &- \sum_{i=1}^{n}\left(\frac{x_i^{ref}}{1+x_i^{ref}} C_{server}^{[i]}\beta_i - \upsilon_i\right)|\bar{x}_i(t)| \\ &+ \sum_{i=1}^{n}\sum_{j\neq i}^{n} \frac{\bar{\gamma}_{ji}}{2}\bar{x}_i^2(t) - \sum_{i=1}^{n}\sum_{j\neq i}^{n} \frac{\bar{\gamma}_{ij}}{2}\bar{x}_j^2(t-\tau_{ij}) \\ \leq& -\sum_{i=1}^{n} \rho_i \bar{x}_i^2(t) - \sum_{i=1}^{n} \nu_i |\bar{x}_i(t)| \leq 0.\end{aligned}$$

In the second step, we have changed the sequence in the summation $\sum_{i=1}^{n}\sum_{j\neq i}^{n}\frac{\bar{\gamma}_{ij}}{2}\bar{x}_j^2(t)$. The above analysis shows that $\lim_{t\to\infty}\sum_i^n |\bar{x}_i(t)| = 0$ [5]. ■

The above stability result belongs to the category of delay-independent result. As revealed in the theorem, our control design is independent of the size of the delay terms, thus provides robustness against arbitrarily large delays. In general, the delay-independent result requires relatively stronger system structural conditions and tighter parameter ranges to guarantee stability robustness against arbitrary delays. However, conservativeness such as waste of control capacity may occur due to the delay-independent design. It can be shown [2] that the requirement in Theorem 2 is stronger than that of Theorem 1, and might be conservative if the network delays in (1) are small. Next, we show that small delays will not destroy the asymptotic regulation result of Theorem 1.

C. Delay-dependent Design for Networks with Delays

We now consider conditions in Assumption 1 again and find upper bounds on the delays such that the asymptotic regulation in Theorem 1 is not destroyed. We introduce: $Q = [q_{ij}]_{n\times n}, i, j = 1, ..., n$, a square matrix.

$$\begin{aligned}q_{ii} =& \frac{x_i^{ref}}{1+x_i^{ref}} C_{server}^{[i]}\alpha_i - \phi_{\tau i}, \quad q_{ij} = \frac{\bar{\gamma}_{ij}+\bar{\gamma}_{ji}}{2}, \\ \phi_{\tau i} =& \frac{1}{2}\sum_{j\neq i}^{n} \tau_{ij}\bar{\gamma}_{ij}\frac{x_j^{ref}C_{server}^{[j]}}{1+x_j^{ref}}\alpha_j + \frac{1}{2}\sum_{j\neq i}^{n}\alpha_i\bar{\gamma}_{ji}\frac{x_i^{ref}C_{server}^{[i]}}{1+x_i^{ref}}\tau_{ji} \\ &+ \frac{1}{2}\sum_{j\neq i}^{n}\sum_{k\neq j}^{n}\tau_{ij}\bar{\gamma}_{ij}\bar{\gamma}_{jk} + \frac{1}{2}\sum_{j\neq i}^{n}\sum_{k\neq j}^{n}\bar{\gamma}_{ji}\bar{\gamma}_{kj}\tau_{kj}, \\ \nu_i =& \frac{x_i^{ref}}{1+x_i^{ref}}C_{server}^{[i]}\beta_i - \upsilon_i - \sum_{j\neq i}^{n}\tau_{ij}\left(\upsilon_j + \frac{x_j^{ref}C_{server}^{[j]}}{1+x_j^{ref}}\beta_j\right).\end{aligned}$$

Note that $\phi_{\tau i} \to 0$ if $\tau_{max} \to 0$.

We are now ready to state our main result for controlling the large-scale network with delays.

Theorem 3: Suppose that in (1), $0 < \tau_{ij} \leq \tau_{max}, 1 \leq i, j \leq n$. Consider the closed-loop large-scale network system (with inter-node delays) composed of (1) and the control law (8). If it holds that the matrix $Q = [q_{ij}]_{n\times n}$ is positive definite and $\nu_i \geq 0,\ \ i = 1, ..., n$, the trajectories of the large-scale network converge asymptotically to $\{X \in \Re^n | x_i = x_i^{ref}, i = 1, ..., n\}$.

Proof: Similarly with Lemma 1, we can show that there exists some finite $T \geq 0$ such that $x_i(t) \in [x_i^{ref}, x_i^*], \forall t \geq T$, $x_i^* = \frac{1-\beta_i}{\alpha_i} + x_i^{ref}$. It follows that $\alpha_i\bar{x}_i(t) + \beta_i sgn(\bar{x}_i(t)) \leq 1$,

$$C_i(x_i(t)) = C_{server}^{[i]}[\alpha_i\bar{x}_i(t) + \beta_i sgn(\bar{x}_i(t))] \ \ \forall i = 1, ..., n.$$

We consider the closed-loop system for $t \geq T + 2\tau_{max} > T$. By applying Leibniz-Newton formula, it can be shown

$$|\bar{x}_j(t-\tau_{ij})| \leq |\bar{x}_j(t)| + \int_{-\tau_{ij}}^{0} C_{server}^{[j]}\cdot\alpha_j\frac{x_j^{ref}}{1+x_j^{ref}}\bar{x}_j(t+\theta)d\theta + \int_{-\tau_{ij}}^{0}\sum_{k\neq j}^{n}\bar{\gamma}_{jk}\bar{x}_k(t-\tau_{jk}+\theta)d\theta + \tau_{ij}(\upsilon_j + C_{server}^{[j]}\frac{x_j^{ref}}{1+x_j^{ref}}\beta_j).$$

The above relation leads to that

$$\begin{aligned}|\bar{x}_i(t)\bar{x}_j(t-\tau_{ij})| \leq& |\bar{x}_i(t)\bar{x}_j(t)| \\ &+ \underbrace{\int_{-\tau_{ij}}^{0} C_{server}^{[j]}\frac{x_j^{ref}}{1+x_j^{ref}}\alpha_j\bar{x}_i(t)\bar{x}_j(t+\theta)d\theta}_{o_1} \\ &+ \underbrace{\int_{-\tau_{ij}}^{0}\sum_{k\neq j}^{n}\bar{\gamma}_{jk}\bar{x}_i(t)\bar{x}_k(t-\tau_{jk}+\theta)d\theta}_{o_2} \\ &+ \tau_{ij}(\upsilon_j + C_{server}^{[j]}\frac{x_j^{ref}}{1+x_j^{ref}}\beta_j)|\bar{x}_i(t)|.\end{aligned}$$

By completing the squares,

$$\begin{aligned}o_1 \leq& \frac{1}{2}\tau_{ij}\frac{x_j^{ref}}{1+x_j^{ref}}C_{server}^{[j]}\alpha_j\bar{x}_i^2(t) \\ &+ \frac{1}{2}\int_{-\tau_{ij}}^{0}\frac{x_j^{ref}}{1+x_j^{ref}}C_{server}^{[j]}\alpha_j\bar{x}_j^2(t+\theta)d\theta, \\ o_2 \leq& \frac{1}{2}\tau_{ij}\sum_{k\neq j}^{n}\bar{\gamma}_{jk}\bar{x}_i^2(t) \\ &+ \frac{1}{2}\int_{-\tau_{ij}}^{0}\sum_{k\neq j}^{n}\bar{\gamma}_{jk}\bar{x}_k^2(t-\tau_{jk}+\theta)d\theta.\end{aligned}$$

It follows from the above steps that

$$\begin{aligned}\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}\bar{\gamma}_{ij}|\bar{x}_i(t)\bar{x}_j(t-\tau_{ij})| \leq& \sum_{i=1}^{n}\sum_{j\neq i}^{n}\bar{\gamma}_{ij}|\bar{x}_i(t)\bar{x}_j(t)| \qquad (13) \\ &+ \frac{1}{2}\sum_{i=1}^{n}\sum_{j\neq i}^{n}\tau_{ij}\bar{\gamma}_{ij}\frac{x_j^{ref}}{1+x_j^{ref}}C_{server}^{[j]}\alpha_j\bar{x}_i^2(t) \\ &+ \frac{1}{2}\sum_{i=1}^{n}\sum_{j\neq i}^{n}\int_{-\tau_{ij}}^{0}\bar{\gamma}_{ij}\frac{x_j^{ref}}{1+x_j^{ref}}C_{server}^{[j]}\alpha_j\bar{x}_j^2(t+\theta)d\theta\end{aligned}$$

$$+\frac{1}{2}\sum_{i=1}^{n}\sum_{j\neq i}^{n}\sum_{k\neq j}^{n}\tau_{ij}\bar{\gamma}_{ij}\bar{\gamma}_{jk}\bar{x}_i^2(t)$$
$$+\frac{1}{2}\sum_{i=1}^{n}\sum_{j\neq i}^{n}\sum_{k\neq j}^{n}\int_{-\tau_{ij}}^{0}\bar{\gamma}_{ij}\bar{\gamma}_{jk}\bar{x}_k^2(t-\tau_{jk}+\theta)d\theta$$
$$+\sum_{i=1}^{n}\sum_{j\neq i}^{n}\tau_{ij}\left(\upsilon_j+\frac{x_j^{ref}}{1+x_j^{ref}}C_{server}^{[j]}\beta_j\right)|\bar{x}_i|.$$

We now analyze the performance of the control system composed of (1) and the control law $C_i(x_i)$. Consider the Lyapunov-Krasovskii functional candidate

$$V = \sum_{i=1}^{n}[\frac{1}{2}\bar{x}_i^2(t)+W_i], \text{ where}$$
$$W_i = \frac{1}{2}\sum_{j\neq i}^{n}\int_{-\tau_{ij}}^{0}\int_{t+\theta}^{t}\bar{\gamma}_{ij}\frac{x_j^{ref}}{1+x_j^{ref}}C_{server}^{[j]}\alpha_j\bar{x}_j^2(s)dsd\theta$$
$$+\frac{1}{2}\sum_{j\neq i}^{n}\sum_{k\neq j}^{n}\int_{-\tau_{ij}}^{0}\int_{t-\tau_{jk}+\theta}^{t}\bar{\gamma}_{ij}\bar{\gamma}_{jk}\bar{x}_k^2(s)dsd\theta.$$

Differentiating V along the trajectories of the closed-loop system leads to:

$$\dot{V}=\sum_{i=1}^{n}\left[\bar{x}_i(t)\left(-\frac{x_i(t)}{1+x_i(t)}C_i(x_i(t))+\lambda_i(t)\right)+\dot{W}_i(t)\right]$$
$$\leq-\sum_{i=1}^{n}\frac{x_i^{ref}C_{server}^{[i]}}{1+x_i^{ref}}\alpha_i\bar{x}_i^2(t)+\sum_{i=1}^{n}\sum_{j\neq i}^{n}\bar{\gamma}_{ij}\bar{x}_i(t)\bar{x}(t-\tau_{ij})$$
$$-\sum_{i=1}^{n}\left(\frac{x_i^{ref}}{1+x_i^{ref}}C_{server}^{[i]}\beta_i-\upsilon_i\right)|\bar{x}_i(t)|+\sum_{i=1}^{n}\dot{W}_i(t).$$

Combining with the previously derived inequality (13), we can arrive at the following inequality after some calculation:

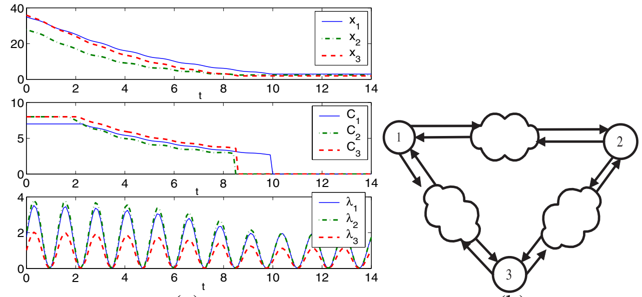
$$\dot{V} \leq -\bar{X}^TQ\bar{X}-\sum_{i=1}^{n}\nu_i|\bar{x}_i(t)|\leq 0.$$

The asymptotic regulation of large-scale networks with delays thus follows [5]. ■

Simulation result

We now present our simulation results for a three-node interconnected system, using the control design presented in Theorem 3. The parameters of the three nodes and their respective controller parameters are shown in the following Table. The uncertain interfering traffic λ_is are modelled by sine waves. We take σ_{ij}s as $\tan^{-1}$ functions. The asymptotic regulations of the three-node network and the structure of interconnections are shown respectively in Figure 1, part (a) and (b).

	node 1	node 2	node 3	γ		τ	
$x_{buffer}^{[i]}$	35	28	36	γ_{12}	2.01	τ_{12}	0.4
$\phi_i(\theta)$	35	28	36	γ_{13}	2.42	τ_{13}	0.2
x_i^{ref}	3	2.5	2	γ_{21}	2.34	τ_{21}	0.4
$C_{server}^{[i]}$	7	8	8	γ_{23}	2.65	τ_{23}	0.5
α_i	0.1	0.08	0.96	γ_{31}	1.4	τ_{31}	0.6
β_i	0.17	0.16	0.25	γ_{32}	2.8	τ_{32}	0.3



(a) (b)

Fig. 1. Three-node interconnected control system performance

IV. Conclusion

Through theoretic analysis and simulations, we have shown that our constrained control law achieves asymptotic regulation for a class of large-scale networks against uncertain time-varying inter-node traffic, transmission delays, and node capacity and buffer size constraints. The problem is solved under conditions motivated by physical characteristics of network traffic (in Assumption 1). The delay-independent control scheme offers stability robustness against arbitrary network delays, but may cause resource waste when delays are small. It is also discovered the decentralized control design in Section III-A (for the case when delays are omitted) is robust against small network delays, using the idea of delay-dependent design.

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