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Constructive Output Feedback AQM Design

Yi Fan[1], Zhong-Ping Jiang[2], Shivendra Panwar[3], Hao Zhang[1], and Xingxing Wu[1]

Electrical and Computer Engineering Department
Polytechnic University
Six Metrotech Center
Brooklyn, NY, 11230, U.S.A.
[1]{yfan01, hzhang08, xwu03}@utopia.poly.edu, [2]zjiang@control.poly.edu, [3]panwar@catt.poly.edu

***Abstract*— A novel constructive output feedback congestion controller is presented for asymptotically stabilizing a class of TCP networks at a desired operating point. An observer for the TCP window size is proposed to aid the task of designing AQM controller with only the output (queue size) measurement. Conditions under which the TCP window estimation converges to the true window size is related to the persistent excitation (PE) condition [9] of the control input. Observer-based backstepping design technique is applied for developing the control law, in the presence of amplitude limits on the control input–packets dropping or marking ratio must fall between 0 and 1. An estimation of the domain of attractiveness is provided by a Lyapunov level set [9].**

***Index Terms*— TCP, Congestion control, AQM, Persistent excitation, Output feedback.**

I. Introduction

Internet congestion control with stability considerations has been addressed in a large body of work. The most prevalent AQM (Active Queue Management) schemes such as drop-tail and Random Early Detection (RED) are based on engineering intuitions. Although they have alleviated congestion under certain situations, their shortcomings are also observed. For example, drop-tail tends to keep a long queue at the bottleneck link buffer and subject instantaneous traffic bursts to high packet-loss [7]. It is difficult to tune the RED parameters to tradeoff between stability and system responsiveness [5][20].

Noticing the defects of the above prevailing schemes, robustness of congestion control algorithms w.r.t. delay, disturbance and uncertainty attracts great attention in the past decade. For example, authors of [5] and [19] use frequency domain tools to analyze the stability margin of the linearized model. Other stability results are obtained using nonlinear and game theoretic analysis, see a recent book [22] and [1][2][8][13][18][23][25][8][26][17] for existing results.

Compared with the available results on stability analysis of the existing congestion control schemes, the synthesis of new controllers with improved performance has not received enough attention, and the body of existing results are relatively small. The authors of [22][27] compared the performance of several existing AQM schemes, but does not systematically address how to design AQMs using control theory.

We focus on the design of new AQMs (controllers implemented at link routers) via control theoretic approaches. We do not change the plant dynamics at the end-host side. The other approach of modifying the TCP protocols of the end-host has been studied in [14][16]. We now introduce a few results relevant to our subject. A state feedback design is proposed in [4] to stabilize the buffer queue length at a bottleneck link. The controller uses both the window size and the queue length measurements. Since the Internet is a large-scale complex system, measuring every window size is not feasible. An output feedback design is more preferable as it requires only limited output information. Hollot and Chait in [7] proposed a static output feedback controller ("proportional marking") to stabilize the TCP/AQM closed-loop system. The equilibrium queue length is characterized by network parameters and is thus not free to choose. The authors of [6] then proposed to apply "PI" controller instead of the "P" controller. The output feedback LQ (Linear-Quadratic) AQM design in [12] is again based on a linear network model. All the above new AQM schemes are linear designs. Analysis for the closed-loop control systems basically involves linearization ideas and leads to local asymptotic stability. [21][24] investigate the interesting direction of applying sliding mode control technique to design AQM algorithm. However, the control law is again developed via state feedback and bears a similar shortcoming as [4]. In summary, the constructive congestion controller design via output feedback to stabilize the nonlinear network model is still largely unexplored, even for the case when feedback delay is omitted.

Inspired by the above discussion, we are interested in developing an output feedback AQM controller for the nonlinear TCP model. As opposed to the linear frequency domain methods used in [4][6][5][28][16][14][15], we apply observer based backstepping design technique and apply Lyapunov's direct method, with the hope to enlarge the domain of stability. To this end, we first present an observer

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for estimating end-host TCP window size. The observer state (window size estimation) is guaranteed to converges to the real state (true window size) asymptotically, under the condition that the control input is "PE". Asymptotic stabilizer is then developed with the window size estimate and the measured output (queue length). One contribution of our work is that we achieve asymptotic stability via output (queue length) feedback instead of state feedback. Secondly, saturation constraint is a challenging issue in nonlinear control systems. It deserves further attention in the design and analysis of congestion control schemes. We address the constraints on the control input-the packets dropping or marking ratio must fall between 0 and 1, and we give an estimation for the domain of stability.

The problem of observer design for nonlinear systems is in general difficult. As the first step toward the control design for a general network, we study in this work the ideal network model (i.e., without delay and disturbances). We will show that our study for the ideal network model also contributes a non-trivial case study in nonlinear control.

II. Problem formulation and hypotheses

The results in this paper are based on the fluid model proposed by Kelly [10], Misra *et al.* [15], and Low *et al.* [11]. The following dynamic equations include both the dynamics of the bottleneck link buffer and the dynamics of the window size, which are typical behaviors of the TCP/AQM network.

$$\dot{q} = \begin{cases} 0 & if\ q = 0\ and\ N\frac{W}{\tau} - C < 0 \\ N\frac{W}{\tau} - C & otherwise \end{cases} \quad (1)$$

$$\dot{W} = \frac{1}{\tau} - \frac{W^2 + 2}{2\tau}p \quad (2)$$

$$q \in [0, q_{max}], p \in [0, 1],$$

where the bottleneck link buffer queue length q and the end host window size W are the state variables. N, C and τ represent the number of users (load factor), link capacity and the round trip time. q_{max} denotes the maximum buffer size. Packet dropping or marking probability p in (2) is the control input to be found. We will design p to meet the requirement such that $p(t) \in [0, 1]$. Equation (1) models the queue accumulation as the integration of the excess of the packets sending rate over the link capacity. Equation (2) models the *Additive-Increase and Multiplicative-Decrease* window size evolution in the congestion avoidance phase of TCP networks. This model considers the situation of multiple homogeneous TCP sources, a single bottleneck-link and a delay-free feedback. Detailed justification of this model can be found in [10], [15].

The design objective is to develop a stabilizing feedback control p for the plant (1)-(2) to achieve the convergence of W to an operating point W^* which fully utilizes the link capacity, and q to a desired length q^*, when only the output–buffer queue length q–is measured, as illustrated in the control system block diagram in Figure 1. We assume N, C, τ are known.

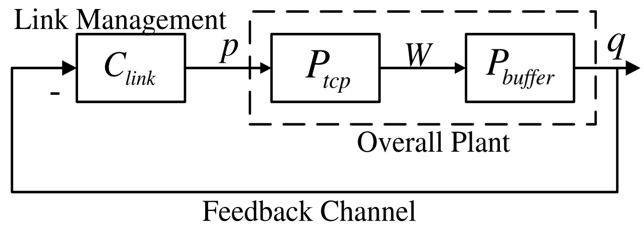

Fig. 1. Control system block diagram.

Our work is based on the following two hypotheses regarding $p(t)$, packets dropping or marking ratio.

Hypothesis 1: There exist $t_0 \geq 0$ (possibly large) and some $T_0 > 0, \alpha_0 > 0$ such that for $\forall t \geq t_0$, p satisfies:

$$\frac{1}{T_0}\int_t^{t+T_0} p(\tau)d\tau \geq \alpha_0. \quad (3)$$

Hypothesis 2: $\limsup_{t\to\infty} p(t) < 1$.

Remark 1: The above hypotheses are motivated by the physical characteristics of the network. In the context of congestion control in TCP networks, the feedback signal p represents packet dropping or marking probability. We interpret this probability as the ratio of packets that are dropped or marked to all packets arriving at bottleneck link buffer. The above requirement (3) is known as a "PE" (persistence of excitation) requirement [9] and is equivalent to that, the overall marked or dropped packets must reach a certain level over every period of time of some length T_0 in the long run, otherwise the link buffer will not be cleared and severe congestion will occur due to accumulated packets. Hypothesis 2 requires that for t large enough, the packet dropping probability is bounded away from 1. It means that dropping too many packets is undesirable and should be avoided in the long run. With such understandings, we believe these two hypotheses are not too restrictive.

III. Main result

A. Observer design.

First consider the following dynamic equation of an open loop observer, where $\widehat{W}$ is the observer state.

$$\dot{\widehat{W}} = \frac{1}{\tau} - \frac{\widehat{W}^2 + 2}{2\tau}p \quad (4)$$

Define

$$\varepsilon \doteq W - \widehat{W} \quad (5)$$

as the estimation error between the real window size and its estimation.

The following lemma is useful for proving that the window size estimation error converges to zero asymptotically.

Lemma 1: Consider the differential equation (4) where $p(t) \in [0,1]$ and satisfies Hypotheses 1, 2. If the initial value satisfies $\widehat{W}(0) \geq 0$, the window size observer state $\widehat{W}(t)$ satisfies that $\widehat{W}(t) \geq 0$, $\forall t \geq 0$ and $\liminf_{t\to\infty} \widehat{W}(t) > 0$.

Proof: See Appendix A. ■

With the help of Hypothesis 1 and the fact that $\liminf_{t\to\infty} \widehat{W}(t) > 0$ established by Lemma 1, we can arrive at the following proposition regarding the convergence of the window size estimation error.

Proposition 1: Suppose the initial value for the observer state is chosen such that $\widehat{W}(0) \geq 0$. For $t \geq t_0$, where $t_0 > 0$ is large enough, the window size estimation error defined by (5) satisfies (exponential convergence)

$$|\varepsilon(t)| \leq m|\varepsilon(t_0)|e^{-\gamma_1(t-t_0)},$$

where $m = e^{\liminf_{t\to\infty} \widehat{W}(t)\frac{\alpha_0 T_0}{2\tau}}$, $\gamma_1 = \frac{\alpha_0}{2\tau}\liminf_{t\to\infty} \widehat{W}(t)$, if input p of (2),(4) is a nonnegative function of t in [0,1], and satisfies Hypotheses 1 and 2.

Proof: See Appendix B. ■

B. Output feedback control design.

With the window size estimation error $\varepsilon(t)$ being an exponentially converging to zero signal when t is large enough, we can consider the following plant.

$$\dot{q} = \begin{cases} 0 & if\ q = 0\ and\ N\frac{W}{\tau} - C < 0, \\ \frac{N}{\tau}(\widehat{W} + \varepsilon) - C & otherwise, \end{cases} \quad (6)$$

$$q \in [0, q_{max}],$$

$$\dot{\widehat{W}} = \frac{1}{\tau} - \frac{\widehat{W}^2 + 2}{2\tau}p\ ,\ p(t) \in [0,1]. \quad (7)$$

In the above equations, variable q and parameters N, τ, C follow the same definitions as in (1)-(2). Equation (6) is obtained from (1) by substituting W with $\widehat{W} + \varepsilon$ using (5). ε represents the estimation error between the window size W and the observer state $\widehat{W}$.

Controller development

In this part, we design a control input $p(t)$ using the output measurement $q(t)$ and the observer state $\widehat{W}(t)$ to stabilize the buffer queue length $q(t)$ at a desired value $q^* > 0$, where q^* is a design freedom.

We use backstepping technique for the control law development. First we consider the subsystem (6). We think of the observer state $\widehat{W}$ as a virtual control input and stabilize q at q^*. We then include the entire plant (6)-(7) and design the real control input p asymptotically tracking $\widehat{W}$ to the ideal form derived in the first step. Saturation constraint on $p(t)$ needs a careful treatment and will be dealt with later. *Step* 1: define $z_1 = q - q^*$. Consider $V_1 = \frac{1}{2}z_1^2$. We differentiate V_1 with respect to time t along the trajectories of z_1-subsystem.

$$\dot{V}_1 = z_1\Big(\frac{N}{\tau}\widehat{W} + \frac{N}{\tau}\varepsilon - C\Big) \leq z_1\Big(\frac{N}{\tau}\widehat{W} + \frac{Nk_1}{4\tau}z_1 - C\Big) + \frac{N}{k_1\tau}\varepsilon^2.$$

We have used completing the squares in the above derivation, namely $\frac{N}{\tau}z_1\varepsilon \leq \frac{Nk_1}{4\tau}z_1^2 + \frac{N}{k_1\tau}\varepsilon^2$. $k_1 > 0$ is a constant parameter.

Let $\widehat{W} = \frac{\tau}{N}z_2 + \alpha_1(z_1)$ where

$$\alpha_1(z_1) = -k_1 z_1 + \frac{\tau C}{N}$$

is the desired feedback law to stabilize the subsystem (6) using the virtual control input $\widehat{W}$. Then

$$z_2 := \frac{N}{\tau}\widehat{W} - \frac{N}{\tau}\alpha(z_1) = \frac{N}{\tau}\widehat{W} + k_1\frac{N}{\tau}z_1 - C$$

contains the error between $\widehat{W}$ and the ideal form $\alpha_1(z_1)$. Substituting

$$\widehat{W} = \frac{\tau}{N}z_2 - k_1 z_1 + \frac{\tau C}{N}$$

into the inequality for $\dot{V}_1$ yields

$$\dot{V}_1 \leq z_1 z_2 - \frac{3N}{4\tau}k_1 z_1^2 + \frac{N}{\tau k_1}\varepsilon^2.$$

Step 2: Consider $V_2 = V_1 + \frac{\rho}{2}z_2^2,\ \rho > 0$. We differentiate V_2 along the trajectories of z_1, z_2-system combining the knowledge of $\dot{V}_1$ and $\dot{z}_2$. It follows from *Step* 1 that the dynamic equation of z_2 is:

$$\dot{z}_2 = \frac{N}{\tau}\dot{\widehat{W}} + k_1\frac{N}{\tau}\dot{z}_1 = \frac{N}{\tau}u + k_1\frac{N^2}{\tau^2}\widehat{W} + k_1\frac{N^2}{\tau^2}\varepsilon - k_1\frac{NC}{\tau}$$

where $u := \frac{1}{\tau} - \frac{\widehat{W}^2 + 2}{2\tau}p$ is introduced for conveniences.

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + \rho z_2\dot{z}_2 \\ &= z_1 z_2 - \frac{3N}{4\tau}k_1 z_1^2 + \frac{N}{\tau k_1}\varepsilon^2 \\ &\quad + \rho z_2\left(\frac{N}{\tau}u + k_1\frac{N^2}{\tau^2}\widehat{W} + k_1\frac{N^2}{\tau^2}\varepsilon - k_1\frac{NC}{\tau}\right) \\ &\leq -\frac{3N}{4\tau}k_1 z_1^2 + \rho z_2\left(\frac{N}{\tau}u + \frac{z_1}{\rho} + k_1\frac{N^2}{\tau^2}\widehat{W} - k_1\frac{NC}{\tau}\right) \\ &\quad + \frac{N}{\tau k_1}\varepsilon^2 + k_1\rho\frac{N^2}{\tau^2}z_2\varepsilon.\end{aligned}$$

By completing the squares,

$$k_1\rho\frac{N^2}{\tau^2}z_2\varepsilon \leq \frac{\rho k_2}{2}z_2^2 + \frac{\rho N^4 k_1^2}{2k_2\tau^4}\varepsilon^2,$$

leading to

$$\begin{aligned}\dot{V} &\leq -\frac{3N}{4\tau}k_1 z_1^2 + \frac{N}{\tau k_1}\varepsilon^2 - \rho k_2 z_2^2 + \frac{\rho k_2}{2}z_2^2 + \frac{\rho N^4 k_1^2}{2k_2\tau^4}\varepsilon^2 \\ &\leq -\frac{3N}{4\tau}k_1 z_1^2 - \frac{\rho k_2}{2}z_2^2 + \left(\frac{N}{\tau k_1} + \frac{\rho N^4 k_1^2}{2k_2\tau^4}\right)\varepsilon^2.\end{aligned}$$

The nominal control

$$u = \frac{\tau}{N}\left(-\frac{z_1}{\rho} - \frac{N^2}{\tau^2}k_1\widehat{W} - k_2 z_2 + k_1\frac{NC}{\tau}\right)$$

is applied in the third step. $k_1 > 0, k_2 > 0$ are constants for tuning the control gain. Define $\gamma_2 := \min\left\{\frac{3N}{2\tau}k_1, \rho k_2\right\}$. It follows that

$$\dot{V} \leq -\gamma_2 V_2 + \left(\frac{N}{\tau k_1} + \frac{\rho N^4 k_1^2}{2k_2\tau^4}\right)\varepsilon^2.$$

Note that the window size estimation error $|\varepsilon(t)| \leq m|\varepsilon(t_0)|e^{-\gamma_1(t-t_0)}$, γ_1, m and ε are defined in Section III-A. By the Comparison Lemma [9], for $\forall t \geq t_0$, we have

$$V_2(t) \leq V_2(t_0)e^{-\gamma_2(t-t_0)} + \frac{c}{\gamma_2 - 2\gamma_1}\left[e^{-2\gamma_1(t-t_0)} - e^{-\gamma_2(t-t_0)}\right],$$

where $c := \left(\frac{N}{\tau k_1} + \frac{\rho N^4 k_1^2}{2k_2\tau^4}\right)m^2\varepsilon^2(t_0)$. The above analysis shows that the control scheme is asymptotically stabilizing. Define

$$g(\gamma_1,\gamma_2) := \frac{1}{\gamma_2 - 2\gamma_1}\left[\left(\frac{2\gamma_1}{\gamma_2}\right)^{\frac{2\gamma_1}{\gamma_2-2\gamma_1}} - \left(\frac{2\gamma_1}{\gamma_2}\right)^{\frac{\gamma_2}{\gamma_2-2\gamma_1}}\right].$$

It can be shown that

$$\frac{1}{\gamma_2 - 2\gamma_1}\left[e^{-2\gamma_1(t-t_0)} - e^{-\gamma_2(t-t_0)}\right] \leq g(\gamma_1,\gamma_2).$$

For all $t \geq t_0$,

$$V_2(t) \leq V_2(t_0) + c \cdot g(\gamma_1,\gamma_2).$$

Suppose it holds

$$c \cdot g(\gamma_1,\gamma_2) < \frac{(q_{max} - q^*)^2}{2}, \quad (8)$$

it is then guaranteed that

$$\frac{1}{2}z_1^2 \leq V_2(t) \leq \frac{(q_{max} - q^*)^2}{2}$$

for $\forall t \geq t_0$. It implies $|z_1| = |q - q^*| \leq q_{max} - q^*$, and hence $q(t) \leq q_{max}$ for $\forall t \geq t_0$. Based on the above derivation, an estimation for the domain of stability is

$$\Omega_c = \left\{V(z_1,z_2) \leq \frac{(q_{max} - q^*)^2}{2} - cg(\gamma_1,\gamma_2)\right\}.$$

The actual control law is obtained from u, as

$$p = \frac{2\tau}{\widehat{W}^2 + 2}\left(\frac{1}{\tau} - k_1 C + \frac{\tau}{N\rho}z_1 + k_1\frac{N}{\tau}\widehat{W} + k_2\frac{\tau}{N}z_2\right). \quad (9)$$

Due to the saturation constraints on p, the parameters k_1, k_2 and ρ need to be carefully designed. The following theorem shows that by tuning ρ, k_1 and k_2 appropriately, it is guaranteed that for all $\{q(t_0), \widehat{W}(t_0)\} \in \Omega_c$, $p(t) \in [0,1], \forall t \geq t_0$.

Main result

In view of the above design process, we state the following theorem as our main result.

Theorem 1: Consider the plant model consisting of (1) and (2), which represent the window size and the bottleneck link buffer queue length dynamics. Apply the control law (9) and set the initial value $\widehat{W}(0) \geq 0$ for the observer state. Suppose that k_1, k_2 and ρ are chosen such that

$$\left(k_1k_2 + \frac{\tau}{\rho N}\right)(q_{max} - q^*) + \frac{\tau}{2}\left(k_1\frac{N}{\tau} + k_2\right)^2 \leq k_1 C + k_2\frac{\tau C}{N}. \quad (10)$$

$$k_1 C + k_2\frac{\tau C}{N} + \frac{\tau q^*}{N\rho} + k_1k_2q^* \leq \frac{1}{\tau}. \quad (11)$$

The trajectories of the closed-loop system converge to $\{q^*, W^* = \frac{\tau C}{N}\}$ asymptotically.

Proof: See Appendix C. ■

The following simulation demonstrate that using the controller we designed, the closed loop system converges to the desired equilibrium asymptotically. The link buffer queue length converges asymptotically to $q^* = 5$. The simulation parameters are as follow:

N= 50, τ = 0.1sec, C = 5000packets/sec
q_{max} = 30packets, packets length= 1000bytes,
segment size = 1000bytes, q^* = 5packets.

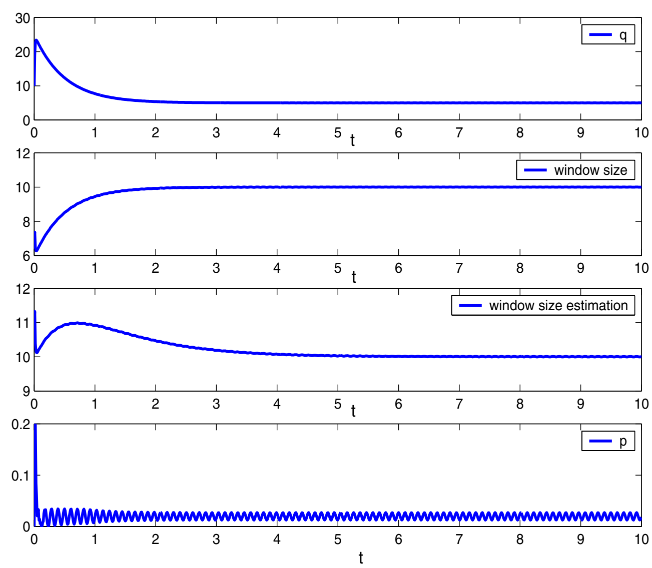

Fig. 2. Closed-loop system response.

Remark 2: Our control law achieves the asymptotic regulation of the output– queue length with Ω_c being an estimation of the region of attractiveness. According to the inequalities (10), (11), performance of the proposed controller is subject to the bandwidth (C) and the round-trip time (τ).

Remark 3: The window size estimation error $W - \widehat{W} \to 0$, as $t \to \infty$, as can be seen by comparing the second and the third plotting of Figure 2.

IV. CONCLUSION

Compared with most existing congestion control schemes, we consider the problem of output feedback control instead of state-feedback control, without using linearization. Using observer based backstepping design method, an output feedback controller is designed for a single bottleneck link network with saturation constraint on the control input and the state. The controller asymptotically stabilizes the system at the desired equilibrium point.

Our future work will be directed to extend the developed output feedback control design to a more practical network model with delays. Nevertheless, as stated previously, our output feedback control scheme to a nonlinear delay-free network model represents a nontrivial application of modern nonlinear control theory. We also believe that the output feedback design advocated here should provide benefits to future networking technical developments and serve as guidance for decentralized protocol designs.

REFERENCES

[1] T. Alpcan and T. Basar, "Global Stability Analysis of End-to-End Congestion Control Schemes for General Topology Networks with Delay," *Proc. 42nd IEEE Conference on Decision and Control*, pp. 1092-1097, Maui, Hawaii, Dec. 2003.

[2] S. Deb and R. Srikant, "Global stability of congestion controllers for the Internet," *IEEE Trans. Automatic Control*, pp. 1055-1060, vol. 48, no. 6, June 2003.

[3] X. Fan, M. Arcak, and J. Wen, " Lp-stability and delay robustness of network flow control," *Proc. 42nd IEEE Conference on Decision and Control*, pp. 3683-3688, Maui, Hawaii, Dec. 2003.

[4] Y. Gao and J. C. Hou , "A state feedback control approach to stabilizing queues for ECN-enabled TCP," *Proc. IEEE INFOCOM*, vol. 3, pp. 2301 - 2311, San Francisco, California, April 2003.

[5] C.V. Hollot, V. Misra, D. Towsley, and W. B. Gong, "A control theoretic analysis of RED", *Proc. IEEE INFOCOM*, pp.1510-1519, Anchorage, Alaska, April 2001.

[6] C.V. Hollot, V. Misra, D. Towlsey, and W. B. Gong, "On designing improved controllers for AQM routers supporting TCP flows," *Proc. IEEE INFOCOM*, vol.3, pp. 1726 - 1734, Anchorage, Alaska, April, 2001.

[7] C.V. Hollot and Y. Chait, "Nonlinear stability analysis for a class of TCP/AQM network", *Proc. 40th IEEE Conference on Decision and Control*, vol. 3, pp. 2309 -2314, Orlando, Florida, December 2001.

[8] R. Johari, D.K.H.Tan, "End-to-end congestion control for the Internet: delays and stability", IEEE/ACM Transactions on Networking, vol. 9, pp. 818 -832, December, 2001.

[9] Khalil, H. K., *Nonlinear Systems*, 3nd edition, Prentice Hall, Upper Saddle River, 2002.

[10] F. P. Kelly, "Mathematical modeling of the Internet," *Proc. 4th International Congress on Industrial and Applied Mathematics*, pp. 105-116, Edinburgh, Scotland, July 1999.

[11] S.H. Low, F. Paganini and J.C. Doyle, "Internet Congestion Control", *IEEE Control Systems Magazine*, pp.28-43, February 2002.

[12] K. B. Kim, A. Tang, and S.H. Low, *"Design of AQM in supporting TCP based on the well-known AIMD model"*, Proc. IEEE Global Telecommunications Conference 2003, GLOBECOM 03, vol. 6, pp. 3226 – 3230, Sanfrancisco, CA, December 2003.

[13] S. Liu, T. Basar, and R. Srikant, "Controlling the Internet: a survey and some new results," Proc. *42nd IEEE Conference on Decision and Control,* vol.3, pp.3048-3057, Maui, Hawaii, December 2003.

[14] S. H. Low, F. Paganini, J. Wang and J. C. Doyle, "Linear stability of TCP/RED and a scalable control", *Computer Networks* vol.43, no. 5, pp. 633-647, Dec. 2003.

[15] V. Misra, W. Gong, and D. Towsley, "A Fluid-based Analysis of a Network of AQM Routers Supporting TCP Flows with an Application to RED", *Proc. of ACM SIGCOMM*, Stockholm, Sweden, September 2000.

[16] F. Paganini, Z. Wang, S.H. Low, and J.C. Doyle, "A new TCP/AQM for stable operation in fast networks", Proc. *IEEE INFOCOM*, vol. 22, no. 1, pp. 96-105, March 2003.

[17] F. Paganini, "A global stability result in network flow control," *Systems and Control Letters*, vol. 46, no. 3, pp. 153-163, 2002.

[18] F. Paganini, J. Doyle, and S. Low, "A control theoretical look at Internet congestion control," in *Multidisciplinary Research in Control: The Mohammed Dahleh Symposium 2002*, Eds. L. Giarre' and B. Bamieh, Lecture Notes in Control and Information Sciences, Springer-Verlag, Berlin, 2003.

[19] P.-F. Quet and H. Ozbay, "On the design of AQM supporting TCP flows using robust control theory," *IEEE Trans. on Automatic Control*, pp.1031 - 1036, vol. 49, no. 6, June 2004.

[20] P. Ranjan, E.H. Abed, and R.J. La, "Nonlinear instabilities in TCP-RED", *Proc. IEEE INFOCOM*, vol. 1, pp. 249 -258, New York, NY, June 2002.

[21] F. Ren, C. Lin, X. Ying, X. Shan, and F. Wang, "A robust active queue management algorithm based on sliding mode variable structure control", *Proc. IEEE INFOCOM*, vol. 1, pp. 13–20, New York, NY April 2002.

[22] R. Srikant, *The Mathematics of Internet Congestion Control*. Birkhauser, Boston, 2004.

[23] J.T. Wen and M. Arcak, "A unifying passivity framework for network flow control", *Proc. IEEE INFOCOM*, vol. 2, pp. 1156 -1166, San Franciso, CA, April 2003.

[24] P. Yan, Y. Gao, and H. Ozbay, Variable structure control in active queue management for TCP with ECN, *Proc. Eighth IEEE International Symposium on Computers and Communication (ISCC 2003)* vol. 2, pp. 1005 – 1011, Kemer - Antalya, Turkey, June 2003.

[25] L. Ying, G. Dullerud, and R. Srikant, "Global stability of Internet congestion controllers with heterogeneous Delays," Proc. *American Control Conference*, pp. 2948-2953, Boston, MA, June 2004.

[26] Z. Wang and F. Paganini, "Global stability with time delay in network congestion control", *Proc. 41st IEEE Conference on Decision and Control*, vol. 3, pp. 2309-2314, Las Vegas, Nevada, December. 2002.

[27] A. Papachristodoulou, L. Li, and J.C Doyle, "Methodological framework for large-scale networl analysis and design," *ACM SIGCOMM Computer Comminications Review,* vol. 34, no. 3, 2004.

[28] L. Zhu and N. Ansari, "Local stability of a new adaptive queue management (AQM) scheme", *IEEE Communications Letters,* vol. 8, pp.406–408, 2004.

APPENDIX

A. Proof of Lemma 1.

Consider the observer dynamics defined by (4). The completeness of the solution of (4) can be established by observing the differential equation and by applying the Comparison Lemma [9]. Since $\widehat{W}(0) \geq 0$ and $\dot{\widehat{W}}(t) \geq -\frac{\widehat{W}^2(t)}{2\tau}$, we have $\widehat{W}(t) \geq 0$ for all $t \geq 0$. For t sufficiently large, by applying Hypothesis 2, $p(t) \leq \bar{p}$ for some $\bar{p} \in (0, 1)$. It holds:

$$\dot{\widehat{W}}(t) \geq \frac{1-\bar{p}}{\tau} - \frac{\widehat{W}^2(t)}{2\tau}\bar{p}.$$

Consider that, the differential equation

$$\dot{y}(t) = \frac{1-\bar{p}}{\tau} - \frac{y^2(t)}{2\tau}\bar{p}$$

has a stable equilibrium at $\sqrt{\frac{2}{\bar{p}}-2} > 0$. By recalling the Comparison Lemma [9], it leads to that $\liminf_{t\to\infty} \widehat{W}(t) > 0$.

B. Proof of Proposition 1.

From the definition of ε in (5), it is easy to see from (2)-(4) that its dynamic equation is

$$\dot{\varepsilon} = \dot{W} - \dot{\widehat{W}} = \frac{1}{2\tau}(\widehat{W}^2 - W^2)\cdot p = -\frac{1}{2\tau}(W+\widehat{W})\cdot\varepsilon\cdot p$$

Consider the closed-form solution for the above differential equation:

$$\varepsilon(t) = \varepsilon(t_0)e^{-\frac{1}{2\tau}\int_{t_0}^{t}\left(W(\nu)+\widehat{W}(\nu)\right)\cdot p(\nu)\cdot d\nu}$$

where $t_0 > 0$ is a large enough value. For all $t \geq t_0$, we can write $t - t_0 = nT_0 + \widetilde{T}$ for some $n \geq 0, T_0 \geq \widetilde{T} \geq 0$. Substitute this relation into the integration $\int_{t_0}^{t} p(\nu)d\nu$ we establish

$$\begin{aligned}\int_{t_0}^{t} p(\nu)d\nu &= \int_{t_0}^{t_0+nT_0+\widetilde{T}} p(\nu)d\nu \\ &= \int_{t_0}^{t_0+T_0} p(\nu)d\nu + \int_{t_0+T_0}^{t_0+2T_0} p(\nu)d\nu \\ &\quad \ldots \int_{t_0+(n-1)T_0}^{t_0+nT_0} p(\nu)d\nu + \int_{t_0+nT_0}^{t_0+nT_0+\widetilde{T}} p(\nu)d\nu \\ &\geq n\alpha_0 T_0 + \int_{t_0+nT_0}^{t_0+nT_0+\widetilde{T}} p(\nu)d\nu \\ &\geq n\alpha_0 T_0,\end{aligned}$$

where the last two inequalities hold because p satisfies the PE condition (3) and is nonnegative. From the conclusion of Lemma 1 and also the fact that the window size $W(t) \geq 0, \forall t \geq 0$[1], the following inequality holds $\forall t \geq t_0$ for some large enough $t_0 > 0$.

$$W(t) + \widehat{W}(t) \geq \widehat{W}(t) \geq \liminf_{t\to\infty} \widehat{W}(t) > 0.$$

Using the above inequality we have:

$$\begin{aligned}\int_{t_0}^{t}\big(W(\nu)+\widehat{W}(\nu)\big)p(\nu)d\nu &\geq \liminf_{t\to\infty}\widehat{W}(t)n\alpha_0 T_0 \\ &\geq \liminf_{t\to\infty}\widehat{W}(t)\alpha_0(t-t_0-T_0).\end{aligned}$$

Now combine the above derivation with the closed-form solution of $\varepsilon(t)$, we know that for $t \geq t_0$:

$$|\varepsilon(t)| \leq m|\varepsilon(t_0)|e^{-\gamma_1(t-t_0)}.$$

[1] Since $W(t)$ is the end host window size, it satisfies $W(0) \geq 0$. Using the same proof as in Lemma 1, we can show that $W(t) \geq 0$, $\forall t \geq 0$.

C. Proof of Theorem 1.

Note that the form of the control law has been developed (see (9)) using Lyapunov's direct method. We now show that given the stated conditions, the control law satisfies the saturation constraints (namely, the parameters in (10) and (11) guarantee $p(t) \in [0,1]$ for all $q \in [0, q_{max}]$ and for all $\widehat{W}$) and is indeed stabilizing. We first show that p is nonnegative, then show it is upper bounded by 1.

Substitute the definition of z_1 and z_2 into (9), it holds:

$$\begin{aligned}&\frac{1}{\tau} - k_1C + \frac{\tau}{N\rho}z_1 + k_1\frac{N}{\tau}\widehat{W}(t) + k_2\frac{\tau}{N}z_2(t) = \\ &\underbrace{\frac{1}{\tau} - k_1C - k_2\frac{\tau C}{N} - \frac{\tau q^*}{N\rho} - k_1k_2q^*}_{\pi_1} + \underbrace{k_1k_2q + \Big(k_1\frac{N}{\tau}+k_2\Big)\widehat{W}}_{\pi_2(t)}.\end{aligned}$$

According to the dynamic equation of q in (6), it satisfies $q(t) \geq 0$ for $\forall t \geq 0$. From Lemma 1 we know that $\widehat{W}(t) \geq 0$. In the above equation $\pi_2(t) \geq 0$. By (11), $\pi_1 \geq 0$. According to the definition of p in (9) and the above analysis for π_1, π_2, we know that $p(t) \geq 0$, $\forall t \geq 0$.

On the other hand, for all $\{q(t_0), \widehat{W}(t_0) \in \Omega\}, q(t) \leq q_{max}, \ \forall t \geq t_0$. Combining with (10), the following inequality holds:

$$\begin{aligned}\left(k_1k_2 + \frac{\tau}{\rho N}\right)(q(t)-q^*) &\leq k_1C + k_2\frac{\tau C}{N} + \frac{1}{2\tau}\widehat{W}^2 \\ &\quad -\Big(k_1\frac{N}{\tau}+k_2\Big)\widehat{W}\end{aligned}$$

by completing the squares. Applying the above inequality, we can verify

$$\frac{1}{\tau} - k_1C + \frac{\tau}{\rho N}z_1 + k_1\frac{N}{\tau}\widehat{W} + k_2\frac{\tau}{N}z_2 \leq \frac{\widehat{W}^2+2}{2\tau},$$

which implies that $p(t) \leq 1, \ \forall t \geq t_0$ by the definition of p (see (9)).

According to Hypotheses 1 and 2, $p(t)$ satisfies (3) and $\limsup_{t\to\infty} p(t) < 1$. Thus $p(t)$ satisfies the conditions required by Proposition 1. It follows that the window size estimation error defined by (5) converges to zero exponentially.

The rest of the proof is clear from the controller synthesis. Suppose that (8) holds, an estimation for the domain of attraction is given by Ω_c.