Nonlinear Output Feedback Control of TCP/AQM Networks

Yi Fan, Zhong-Ping Jiang, Shivendra Panwar, and Hao Zhang

Abstract— **An output-feedback Active Queue Management (AQM) scheme is presented for asymptotically stabilizing a class of TCP networks at a desired operating point. In order to design an AQM controller with only an output (queue size) measurement, a novel TCP window size observer is first proposed. Then, the control law is developed by applying an observer-based backstepping design technique. The amplitude limits on the control input is addressed, namely, packet dropping or marking ratio must fall between 0 and 1. An estimation of the domain of attractiveness is given with a Lyapunov level set [3].**

Index Terms— **TCP, Congestion control, AQM, Persistent excitation, Output feedback.**

I. Introduction

Most previous efforts on studying Internet congestion control have the aim of analyzing the stability robustness of existing AQM (Active Queue Management) algorithms. See for example, [7][9][8] and a recent book [6]. The development of improved AQM controllers, in contrast, has not received enough attention, and the body of literature is relatively small.

Since the inefficiency of the prevalent AQMs (eg. Droptail and RED) are already reported [8], we focus here on the design of new AQM schemes (controllers operating on links) via control theoretic approaches, while we do not change the plant dynamics at the end-host side. A few relevant results with our subject are as follows. A state feedback design is proposed in [1] to stabilize the TCP/AQM closed loop system. The controller uses both the window size and the queue length measurements. An output feedback design is more preferable as it requires only limited output information. A static output feedback controller ("proportional marking") and a "PI" controller is applied in [2]. The output feedback LQ (Linear-Quadratic) AQM design in [4] is based on a linear network model. All the above new AQM schemes are simple linear designs. Analysis for the closed-loop control systems basically involves linearization ideas and leads to local asymptotic stability. The authors of [8] investigate the interesting idea of applying sliding mode control to stabilize TCP. However, the control law is again developed via state feedback and bears similar shortcomings as [1]. In summary, the constructive congestion controller design via output feedback to stabilize the nonlinear network model is still largely unexplored, even for the case when network delays are omitted.

The focus of our attention is on developing output feedback AQM controllers for the nonlinear TCP model. As opposed to the linear frequency domain methods used in most previous work, we apply observer based backstepping design technique and Lyapunov's direct method [3] for theoretical analysis, with the hope to enlarge the domain of stability. To this end, we first study the observer design for estimating the end-host TCP window size. The observer state (window size estimation) is guaranteed to converge to the real state (true window size) asymptotically, under the condition that the control input is "persistently exciting" (PE) [3]. An asymptotic stabilizer is then developed with the window size estimation and the measured output queue length. One contribution of our work is that we achieve the asymptotic stability for the nonlinear TCP/AQM system via output (queue length) feedback. Secondly, the saturation constraint is a challenging issue in control systems. We address the constraint on the control input: the packets dropping or marking ratio must fall between 0 and 1. We also give an estimation for the domain of stability.

The problems of observer design for nonlinear systems are in general difficult. As the first step toward the control design for a general network, we study the ideal network model (i.e., without delays and disturbances). We will show that our study for the ideal network model also constitute a non-trivial case study in nonlinear control.

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II. Dynamic model and design objective

The results in this paper are based on the fluid model proposed by Misra *et al.* [5]. The following dynamic equations include both the dynamics of the bottleneck link buffer and the dynamics of the window size, which are typical behaviors of the TCP/AQM network.

$$\dot{q} = \begin{cases} 0 & , \ if\ q = 0\ and\ N\frac{W}{\tau} - C < 0 \\ N\frac{W}{\tau} - C & , \ otherwise \end{cases} , \quad (1)$$

$$\dot{W} = \frac{1}{\tau} - \frac{W^2+2}{2\tau}p , \quad (2)$$

$$q \in [0, q_{max}], p \in [0,1].$$

The bottleneck link queue length q and the end host window size W are taken as the state variables. p, the packet dropping or marking ratio, is the control input. q is the output of interest. N, C and τ represent the number of users (load factor), link capacity and the round trip time. q_{max} denotes the maximum buffer size. We assume N, C, τ are known.

The control objective is to design a stabilizing feedback control p for the plant (1)-(2) to achieve the convergence of W to an operating point W^* which fully utilizes the link capacity, and the convergence of q to a desired length q^*, when only the buffer queue length q is measured.

Equation (1) models the queue accumulation as the integration of the excess of the packets sending rate over the link capacity. Equation (2) models the *additive-increase and multiplicative-decrease* window size evolution in the congestion avoidance phase of TCP. This model considers the situation of multiple homogeneous TCP sources, a single bottleneck link and a delay free feedback. A detailed justification of this model can be found in [5].

III. Output feedback AQM design

A. Observer design and estimation error convergence

First consider the following dynamic equation of an open loop observer, where $\widehat{W}$ is the observer state.

$$\dot{\widehat{W}} = \frac{1}{\tau} - \frac{\widehat{W}^2+2}{2\tau}p. \tag{3}$$

$$\text{Define } \varepsilon := W - \widehat{W} \tag{4}$$

as the error between the real and estimated window size.

Our work is based on the following two hypotheses regarding the packet dropping signal $p(t)$.

Hypothesis 1: There exist $t_0 \geq 0$ (possibly large) and some $T_0 > 0, \alpha_0 > 0$ such that for $\forall t \geq t_0$, p satisfies:

$$\frac{1}{T_0}\int_t^{t+T_0} p(\tau)d\tau \geq \alpha_0. \tag{5}$$

Hypothesis 2: $\limsup_{t\to\infty} p(t) < 1$.

Remark 1: The above hypotheses are motivated by considering the physical characteristics of the network. In the context of congestion control in TCP networks, the feedback signal p represents packet dropping or marking probability. We interpret this probability as the portion of packets that are dropped or marked out of all received packets. The above requirement (5) is known as "persistence of excitation" (PE) requirement and is equivalent to that the overall marked or dropped packets must reach a certain level over every period of time of some length T_0 in the long run, otherwise the link buffer will not be cleared and congestion will occur due to accumulated packets. Hypothesis 2 requires that for t large enough, the packet dropping probability is bounded away from 1. It means that dropping too many packets is undesirable and should be avoided in the long run. We believe these two hypotheses are not too restrictive.

The following lemma is useful for proving that the window size estimation error converges exponentially.

Lemma 1: Suppose the time-varying input signal $p(t)$ belongs to $[0,1]$ and satisfies Hypotheses 1 and 2. If the initial value satisfies $\widehat{W}(0) \geq 0$, the window size observer state $\widehat{W}(t)$ satisfies that $\widehat{W}(t) \geq 0, \forall t \geq 0$ and $\liminf_{t\to\infty} \widehat{W}(t) > 0$.

Proof: Consider the observer dynamics defined by (3). The completeness of the solution of (3) can be established by observing the differential equation and by applying the Comparison Lemma [3]. Since $\widehat{W}(0) \geq 0$ and $\dot{\widehat{W}}(t) \geq -\frac{\widehat{W}^2(t)}{2\tau}$, we have $\widehat{W}(t) \geq 0$ for all $t \geq 0$. Since Hypothesis 2 holds, for t sufficiently large, $p(t) \leq \bar{p}$ for some $\bar{p} \in (0,1)$. It holds:

$$\dot{\widehat{W}}(t) \geq \frac{1-\bar{p}}{\tau} - \frac{\widehat{W}^2(t)}{2\tau}\bar{p}.$$

Consider that, the differential equation

$$\dot{y} = \frac{1-\bar{p}}{\tau} - \frac{y^2(t)}{2\tau}\bar{p}$$

has a stable equilibrium at $\sqrt{\frac{2}{\bar{p}}-2} > 0$. By recalling the Comparison Lemma [3], it leads to that $\underline{\lim}_{t\to\infty}\widehat{W}(t) > 0$. ■

With the help of Hypothesis 1 and the fact that $\liminf_{t\to\infty}\widehat{W}(t) > 0$ (see Lemma 1), we can arrive at the following proposition regarding the convergence of the window size estimation error. The fact that $\widehat{W}(t) \geq 0$ for all $t \geq 0$ is useful later in the control design.

Proposition 1: If the input $p(t) \in [0,1]$ satisfies Hypotheses 1 and 2 and if $\widehat{W}(0) \geq 0$, the window size estimation error defined by (4) converges to 0 asymptotically.

Proof: From the definition of ε in (4), it is easy to see from (2)-(3) that its dynamic equation is

$$\dot{\varepsilon} = \dot{W} - \dot{\widehat{W}} = \frac{1}{2\tau}(\widehat{W}^2 - W^2)\cdot p = -\frac{1}{2\tau}(W+\widehat{W})\cdot\varepsilon\cdot p.$$

Consider the closed-form solution of the above equation:

$$\varepsilon(t) = \varepsilon(t_0)e^{-\frac{1}{2\tau}\int_{t_0}^t \left(W(\nu)+\widehat{W}(\nu)\right)\cdot p(\nu)\cdot d\nu}$$

where $t_0 > 0$ is a large enough value. For all $t \geq t_0$, we write $t - t_0 = nT_0 + \widetilde{T}$ for some $n \geq 0, T_0 \geq \widetilde{T} \geq 0$. Substituting this relation into the integration $\int_{t_0}^t p(\nu)d\nu$ leads to

$$\begin{aligned}\int_{t_0}^t p(\nu)d\nu &= \int_{t_0}^{t_0+nT_0+\widetilde{T}} p(\nu)d\nu \\ &= \int_{t_0}^{t_0+T_0} p(\nu)d\nu + \int_{t_0+T_0}^{t_0+2T_0} p(\nu)d\nu \ldots \\ &\quad + \int_{t_0+(n-1)T_0}^{t_0+nT_0} p(\nu)d\nu + \int_{t_0+nT_0}^{t_0+nT_0+\widetilde{T}} p(\nu)d\nu \\ &\geq n\alpha_0 T_0 + \int_{t_0+nT_0}^{t_0+nT_0+\widetilde{T}} p(\nu)d\nu \\ &\geq n\alpha_0 T_0,\end{aligned}$$

where the last two inequalities hold because p satisfies the PE condition (5) and is nonnegative. From the conclusion of Lemma 1 and also the fact that the window size $W(t) \geq 0, \forall t \geq 0$[1], the following inequality holds $\forall t \geq t_0$ for some large enough $t_0 > 0$.

$$W(t) + \widehat{W}(t) \geq \widehat{W}(t) \geq \liminf_{t\to\infty}\widehat{W}(t) > 0.$$

Using the above inequality we have:

$$\begin{aligned}\int_{t_0}^t \left(W(\nu)+\widehat{W}(\nu)\right)p(\nu)d\nu &\geq \liminf_{t\to\infty}\widehat{W}(t)n\alpha_0 T_0 \\ &\geq \liminf_{t\to\infty}\widehat{W}(t)\alpha_0(t-t_0-T_0).\end{aligned}$$

[1]Since $W(t)$ is the end host window size, it satisfies $W(0) \geq 0$. Using the same proof as in Lemma 1, we can show that $W(t) \geq 0$, $\forall t \geq 0$.

Now combining the above derivation with the closed-form solution of $\varepsilon(t)$, we know that for $t \geq t_0$:

$$\varepsilon(t) \leq m\varepsilon(t_0)e^{-\gamma_1(t-t_0)}$$

where $m = e^{\liminf_{t\to\infty} \widehat{W}(t)\frac{\alpha_0 T_0}{2\tau}}$, $\gamma_1 = \frac{\alpha_0}{2\tau}\liminf_{t\to\infty}\widehat{W}(t)$. α_0, T_0 are as defined in (5). ∎

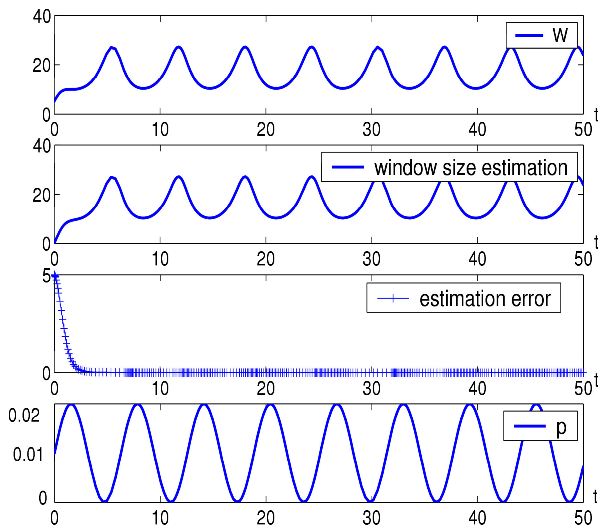

Fig. 1. Convergence of window size estimation error.

The simulation in Figure 1 shows that if $p(t) \in [0,1]$ is a PE signal (We set $p(t)$ to a non-negative sine wave, which satisfies (5)), the estimation error $\varepsilon(t)$ converges to 0 asymptotically.

B. *Control design using output q and observer state $\widehat{W}$*

With the window size estimation error $\varepsilon(t)$ being an exponentially converging to zero signal when t is large enough, we can consider the following model:

$$\dot{q} = \begin{cases} 0 \;, & if\ q=0\ and\ N\frac{W}{\tau} - C < 0 \\ \frac{N}{\tau}(\widehat{W}+\varepsilon) - C \;, & otherwise \end{cases}, \quad (6)$$

$$q \in [0, q_{max}]$$

$$\dot{\widehat{W}} = \frac{1}{\tau} - \frac{\widehat{W}^2+2}{2\tau}p \quad . \quad (7)$$

In the above equations, variable q and parameters N, τ, C follow the same definitions as in (1)-(2). Equation (6) is obtained from (1) by substituting W with $\widehat{W}+\varepsilon$ using (4). ε represents the estimation error between the window size W and the observer state $\widehat{W}$.

Controller synthesis

In this part, we design a control input $p(t)$ using the output measurement $q(t)$ and the observer state $\widehat{W}(t)$ to stabilize the buffer queue length $q(t)$ at a desired value $q^* \in (0, q_{max})$, where q^* is a design freedom.

We apply the backstepping technique and Lyapunov's direct method to complete the design. For simplicity, the boundary effect of (6) is omitted. We consider the system on $[t_0, \infty)$ where t_0 is a large enough value.

Step 1. We first consider the subsystem (6). The observer state $\widehat{W}$ is regarded as a virtual control input to stabilize q at q^*. To this purpose, introduce $z_1 := q - q^*$. Consider the function $V_1 = \frac{1}{2}z_1^2$. Differentiating V_1 to t leads to

$$\dot{V}_1 = z_1\left(\frac{N}{\tau}\widehat{W} + \frac{N}{\tau}\varepsilon - C\right) \leq z_1 z_2 - \frac{3N}{4\tau}k_1 z_1^2 + \frac{N}{\tau k_1}\varepsilon^2,$$

where we have applied

$$\begin{aligned} \widehat{W} &= \alpha(z_1) + \frac{\tau}{N}z_2,\ \alpha(z_1) := -k_1 z_1 + \frac{\tau C}{N}, \\ z_2 &:= \frac{N}{\tau}\widehat{W} - \frac{N}{\tau}\alpha(z_1) = \frac{N}{\tau}\widehat{W} + k_1\frac{N}{\tau}z_1 - C \quad (8) \end{aligned}$$

contains the error between $\widehat{W}$ and its desired form $\alpha(z_1)$. In the above derivation, $\frac{N}{\tau}z_1\varepsilon \leq \frac{Nk_1}{4\tau}z_1^2 + \frac{N}{\tau k_1}\varepsilon^2$ by completing the squares. k_1 is a constant design parameter for tuning the control gain.

Step 2. We then include the entire plant (6)-(7) and use the real control p to asymptotically converge $\widehat{W}$ to its desired form. Notice that from (8)

$$\dot{z}_2 = \frac{N}{\tau}\dot{\widehat{W}} + k_1\frac{N}{\tau}\dot{z}_1 = \frac{N}{\tau}u + k_1\frac{N^2}{\tau^2}\widehat{W} + k_1\frac{N^2}{\tau^2}\varepsilon - k_1\frac{NC}{\tau}$$

where $u := \frac{1}{\tau} - \frac{\widehat{W}^2+2}{2\tau}p$ is introduced for conveniences. Consider the function $V = V_1 + \frac{\rho}{2}z_2^2, \rho > 0$. Along solutions of the control system, the derivative of V_2 to t satisfies:

$$\begin{aligned} \dot{V}_2 =& \dot{V}_1 + \rho z_2 \dot{z}_2 \\ =& z_1 z_2 - \frac{3N}{4\tau}k_1 z_1^2 + \frac{N}{\tau k_1}\varepsilon^2 \\ &+ \rho z_2\left(\frac{N}{\tau}u + k_1\frac{N^2}{\tau^2}\widehat{W} + k_1\frac{N^2}{\tau^2}\varepsilon - k_1\frac{NC}{\tau}\right) \\ \leq& -\frac{3N}{4\tau}k_1 z_1^2 + \rho z_2\left(\frac{N}{\tau}u + \frac{z_1}{\rho} + k_1\frac{N^2}{\tau^2}\widehat{W} - k_1\frac{NC}{\tau}\right) \\ &+ \frac{N}{\tau k_1}\varepsilon^2 + k_1\rho\frac{N^2}{\tau^2}z_2\varepsilon \\ \leq& -\frac{3N}{4\tau}k_1 z_1^2 + \frac{N}{\tau k_1}\varepsilon^2 - \rho k_2 z_2^2 + \frac{\rho k_2}{2}z_2^2 + \frac{\rho N^4 k_1^2}{2k_2\tau^4}\varepsilon^2 \\ \leq& -\frac{3N}{4\tau}k_1 z_1^2 - \frac{\rho k_2}{2}z_2^2 + \left(\frac{N}{\tau k_1} + \frac{\rho N^4 k_1^2}{2k_2\tau^4}\right)\varepsilon^2 \\ \leq& -\gamma_2 V_2 + \left(\frac{N}{\tau k_1} + \frac{\rho N^4 k_1^2}{2k_2\tau^4}\right)\varepsilon^2, \end{aligned}$$

where by completing the squares, $k_1\rho\frac{N^2}{\tau^2}z_2\varepsilon \leq \frac{\rho k_2}{2}z_2^2 + \frac{\rho N^4 k_1^2}{2k_2\tau^4}\varepsilon^2$. The nominal control $u = \frac{\tau}{N}\left(-\frac{z_1}{\rho} - \frac{N^2}{\tau^2}k_1\widehat{W} - k_2 z_2 + k_1\frac{NC}{\tau}\right)$ is applied in the third step. $k_1 > 0, k_2 > 0$ are constants for tuning the control gain. $\gamma_2 := \min\left\{\frac{3N}{2\tau}k_1, \rho k_2\right\}$. Note that the window size estimation error $\varepsilon(t) \leq m\varepsilon(t_0)e^{-\gamma_1(t-t_0)}$. By the Comparison Lemma [3],

$$V_2(t) \leq V_2(t_0)e^{-\gamma_2(t-t_0)} + \frac{c}{\gamma_2 - 2\gamma_1}\left[e^{-2\gamma_1(t-t_0)} - e^{-\gamma_2(t-t_0)}\right],$$

where $c := \left(\frac{N}{\tau k_1} + \frac{\rho N^4 k_1^2}{2k_2\tau^4}\right)m^2\varepsilon^2(t_0)$. The above analysis shows that the control scheme is asymptotically stabilizing.

Suppose that $\varepsilon(t_0)$ satisfies

$$c \cdot g(\gamma_1, \gamma_2) < \frac{(q_{max} - q^*)^2}{2}, \quad (9)$$

$$g(\gamma_1, \gamma_2) := \frac{1}{\gamma_2 - 2\gamma_1}\left[\left(\frac{2\gamma_1}{\gamma_2}\right)^{\frac{2\gamma_1}{\gamma_2 - 2\gamma_1}} - \left(\frac{2\gamma_1}{\gamma_2}\right)^{\frac{\gamma_2}{\gamma_2 - 2\gamma_1}}\right].$$

It follows, an estimation for the domain of stability is

$$\Omega_c = \left\{V(z_1, z_2) \leq \frac{(q_{max} - q^*)^2}{2} - cg(\gamma_1, \gamma_2)\right\},$$

and $q(t) \leq q_{max}, \forall t \geq t_0$. The actual control law is obtained from u, as

$$p = \frac{2\tau}{\widehat{W}^2 + 2}\Big(\frac{1}{\tau} - k_1 C + \frac{\tau}{N\rho} z_1 + k_1 \frac{N}{\tau}\widehat{W} + k_2 \frac{\tau}{N} z_2\Big). \quad (10)$$

Due to the saturation constraints on p, the parameters k_1, k_2 and ρ need to be carefully designed.

Main result

In view of the above design process, we state the following theorem as our main result.

Theorem 1: Consider the plant model consisting of (1) and (2), which represent the window size and the bottleneck link buffer queue length dynamics. Apply the control law (10) and set the initial value $\widehat{W}(0) \geq 0$ for the observer state. Suppose that k_1, k_2 and ρ are chosen such that

$$\left(k_1 k_2 + \frac{\tau}{\rho N}\right)(q_{max} - q^*) \leq k_1 C + k_2 \frac{\tau C}{N} - \frac{\tau}{2}\Big(k_1 \frac{N}{\tau} + k_2\Big)^2. \quad (11)$$

$$k_1 C + k_2 \frac{\tau C}{N} + \frac{\tau q^*}{N\rho} + k_1 k_2 q^* \leq \frac{1}{\tau}. \quad (12)$$

The trajectories of the closed-loop system converge to $\{q^*, W^* = \frac{\tau C}{N}\}$ asymptotically.

Proof: Note that the form of the control law has been developed (see (10)) using Lyapunov's direct method. We now show that given the stated conditions, the control law satisfies the saturation constraints (namely, the parameters in (11) and (12) guarantee $p(t) \in [0, 1]$ for all $q \in [0, q_{max}]$ and for all $\widehat{W}$) and is indeed stabilizing. We first show that p is nonnegative, then show it is upper bounded by 1.

Substitute the definition of z_1 and z_2 into (10), it holds:

$$\frac{1}{\tau} - k_1 C + \frac{\tau}{N\rho} z_1 + k_1 \frac{N}{\tau}\widehat{W}(t) + k_2 \frac{\tau}{N} z_2(t) =$$

$$\underbrace{\frac{1}{\tau} - k_1 C - k_2 \frac{\tau C}{N} - \frac{\tau q^*}{N\rho} - k_1 k_2 q^*}_{\pi_1} + \underbrace{k_1 k_2 q + \left(k_1 \frac{N}{\tau} + k_2\right)\widehat{W}}_{\pi_2(t)}.$$

According to the dynamic equation of q in (6), it satisfies $q(t) \geq 0$ for $\forall t \geq 0$. From Lemma 1 we know that $\widehat{W}(t) \geq 0$. In the above equation $\pi_2(t) \geq 0$. By (12), $\pi_1 \geq 0$. According to the definition of p in (10) and the above analysis for π_1, π_2, we know that $p(t) \geq 0$, $\forall t \geq 0$.

On the other hand, from (11), the following inequality holds:

$$\left(k_1 k_2 + \frac{\tau}{\rho N}\right)(q(t) - q^*) \leq k_1 C + k_2 \frac{\tau C}{N} + \frac{1}{2\tau}\widehat{W}^2 - \left(k_1 \frac{N}{\tau} + k_2\right)\widehat{W}$$

by completing the squares. Applying the above inequality, we can verify

$$\frac{1}{\tau} - k_1 C + \frac{\tau}{\rho N} z_1 + k_1 \frac{N}{\tau}\widehat{W} + k_2 \frac{\tau}{N} z_2 \leq \frac{\widehat{W}^2 + 2}{2\tau},$$

which implies that $p(t) \leq 1$ by the definition of p (see (10)).

According to Hypotheses 1 and 2, $p(t)$ satisfies (5) and $\limsup_{t\to\infty} p(t) < 1$. Thus $p(t)$ satisfies the conditions required by Proposition 1. It follows that the window size estimation error defined by (4) converges to zero exponentially.

The rest of the proof is clear from the controller synthesis. Suppose that (9) holds, an estimation for the domain of attraction is given by Ω_c. ■

The following simulation demonstrate that using the controller we designed, the closed-loop system trajectories converge to the desired equilibrium asymptotically. The link buffer queue length converges asymptotically to $q^* = 5$. The simulation parameters are as follow:

$N = 5$, $\tau = 0.01 sec$, $C = 5000 packets/sec$
$q_{max} = 30 packets$, packets length= 1000bytes,
segment size = 1000bytes, $q^* = 5 packets$.

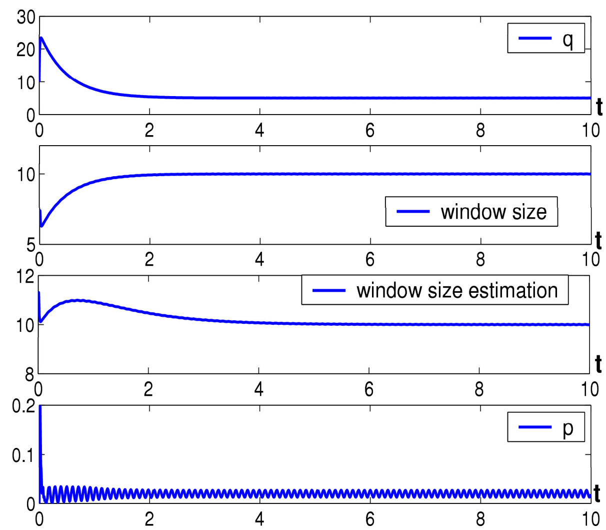

Fig. 2. Closed loop system response.

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