

Collision Resolution Algorithms for a Time-Constrained Multiaccess Channel

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Abstract—Collision resolution algorithms (CRA's) for the ternary feedback multiple access channel with time constraints are considered. We wish to maximize the number of packets successfully transmitted within a fixed deadline K after their arrival for transmission. Packet arrivals are assumed to be Poisson. A nonnested CRA is described and its performance is compared with a nested CRA for values of $K \leq 4$.

I. INTRODUCTION

THE problem of a large number of occasionally active users attempting to share a common channel is referred to as the "multiaccess communications problem" [1]–[3]. Whenever two or more users attempt to transmit a packet at the same time a collision results. When this happens, a collision resolution algorithm (CRA) is used to control retransmissions so that they are eventually transmitted successfully. The objective of these algorithms has been to maximize the throughput of the channel while maintaining a finite average delay. A different objective is necessary when considering time-constrained traffic, such as packetized voice and distributed sensor networks. For such time-constrained traffic, a packet has to be transmitted within a certain time, called its "deadline," or is considered useless and is lost. In such an environment, it is desirable to devise CRA's that minimize the packet loss for a given arrival rate. In this paper, we devise a nonnested CRA for a time-constrained multiaccess channel. We also compare the performance of the nonnested CRA with a nested CRA.

II. MODEL

A standard model for study in the field of multiple access communications has been the slotted channel with ternary feedback [1]. The channel is slotted and each slot corresponds to the packet length. Full sensing and ternary feedback is assumed. That is, at the end of each slot, every user, whether it has a packet to transmit or not, is told and has to keep track of whether the slot was idle, had a successful transmission, or experienced a collision between two or more packets. The distribution of the total traffic generated by all the users is assumed to be Poisson. Several CRA's have been proposed for this channel (e.g., [1], [2]). We will assume,

Paper approved by D. Kazakos, the Editor for Random Access and Distributed Communications Systems of the IEEE Communications Society. Manuscript received January 24, 1990; revised June 18, 1991, and December 9, 1991. This work was supported by the Center for Advanced Technology in Telecommunications, Polytechnic University, Brooklyn, NY, and also in part under NSF Grant NCR-8909719. This paper was presented in part at the Twenty-Fifth Annual Allerton Conference on Communication, Control, and Computing, September 30–October 2, 1987, Urbana-Champaign, IL.

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IEEE Log Number 9211124.

in addition, that all packets have identical deadlines from their time of arrival at the users for transmission. If the packet does not successfully commence transmission within its deadline, it is considered lost. Kurose *et al.* [3] have studied a time constrained carrier-sense multiple-access with collision detection (CSMA/CD) channel and computed the loss for different time constraints. A window algorithm similar to the one used in [1] is employed. An approximate M/G/1 queue model is then used to compute customer losses. In [4], Paterakis *et al.* analyzed a CRA for time-constrained channel with binary (collision–noncollision) feedback. For the case of variable deadlines and the CSMA channel, the effect of different CRA's and transmission policies was studied in [5] and [6] by performing simulations. This paper describes results for both, the nested CRA described in [7] and a nonnested CRA which leads to small improvements over the nested CRA.

III. DESCRIPTION OF CRA

Let all the packet arrivals have a deadline of K time units, where K is a positive real and a time unit corresponds to the length of a slot. This deadline is the maximum allowable delay from the packet's time of arrival to the commencement of its successful transmission. Packets not transmitted within this deadline are lost. Let the total arrival rate be λ per unit time and the percentage loss be $L(\lambda, K)$. We wish to devise CRA's that minimize $L(\lambda, K)$. Let the current time at the beginning of a slot be t_{CT} . Then a window CRA need only enable intervals in the time interval $[t_{CT} - K, t_{CT}]$, since arrivals before $t_{CT} - K$ will already have been lost. For such a model, it is easy to determine the optimal CRA for $K \leq 1$ [7]. Therefore, we will consider the more interesting case of $K > 1$.

For values of $K > 1$ the optimal CRA could not be determined. We develop a CRA that exhibits the first-come, first-served property, since this tends to minimize the number of packets lost [3]. This CRA ensures that packets are transmitted in order of generation. To compute the performance of the CRA's we use the value iteration method [8] because it is a way of determining the optimal CRA within a class of CRA's [2]. This method is briefly summarized next.

Consider a finite-state, discrete-time, ergodic Markov chain. After each transition, the system is in one of N states i , $i = 1, 2, \dots, N$. For each state i , an action $k = 1, 2, \dots, K_i$ is chosen. Then p_{ij}^k is the probability of transition to state j if the process is in state i and action k was chosen. Associated with each transition from i to j , under action k , is a reward r_{ij}^k . Let the value function v_i^n be the total expected reward from the next n transitions, if the system is now in state i , and if an optimal policy is followed. The expected value can be writ-

ten as

$$v_i^{n+1} = \max_{1 \leq k \leq K} \left[\sum_{j=1}^N p_{ij}^k \{r_{ij}^k + v_j^n\} \right], \quad i = 1, 2, \dots, N. \quad (1)$$

Let g be the maximal expected gain per transition. The principal goal in Markov decision processes is to compute g and to find the policy that achieves g . This can be done using the following theorem due to Odoni [9].

Theorem: Let all stationary policies have transition probability matrices representing single-chain aperiodic Markovian processes. Define x_i^n by

$$x_i^n = v_i^{n+1} - v_i^n. \quad (2)$$

Then for any choice of boundary conditions v_i^0 ,

- 1) $x_i^n \rightarrow g$, $1 \leq i \leq N$.
- 2) $L_{\max}^n = \max_i x_i^n$ is monotonically increasing in n to g .
- 3) $L_{\min}^n = \min_i x_i^n$ is monotonically increasing in n to g .
- 4) Any policy π achieving the N maxima in (1) for all n greater than or equal to some n_0 has the maximal gain per transition.

The value iteration algorithm proceeds as follows. The value functions v_i^n , $n = 1, 2, \dots$, are computed using (1). This computation is carried out until $L_{\max}^n - L_{\min}^n < \epsilon$, where ϵ is the maximum allowable error in the computation of g . Then $g \approx (1/2)(L_{\max}^n + L_{\min}^n)$, since $L_{\min}^n \leq x_i^n \leq L_{\max}^n$ from the theorem. The optimal policy that achieves the maximal gain g is the one that achieves the N maxima in (1) for all $n \geq n_0$.

We applied this method to determine the operation of the CRA and to compute the losses. The CRA used is in the spirit of the "part and try" type of ternary tree protocol with an epoch mechanism, as in [1] and [2]. In a manner similar to that described in [1] and [2], three types of states are generated. However, due to the presence of time constraints, the relative position of the sliding window with respect to current time (t_{CT}) has to be included in the state description. The optimal initial window size and the splitting of the collision intervals turn out to be dependent on how far the window is behind current time. An additional complication is that part of the window may be "left behind," that is, a part or an entire window under the process of being resolved may fall more than K time units behind current time. As a result, the arrivals in this window are lost. The three classes of states are described next (also see Fig. 1).

$S_0[h]$: In this state there are no untransmitted arrivals in the time interval $(t_{CT} - K, t_{CT} - h)$, while nothing more than the *a priori* information is known about the time interval $(t_{CT} - h, t_{CT})$, $0 \leq h \leq K$.

$S_1[h, m, r]$: In this state there are no untransmitted arrivals in the time interval $(t_{CT} - K, t_{CT} - h)$. The time interval $(t_{CT} - h, t_{CT} - h + m)$ is part of an interval of length $(m + r)$ in which we know that there is at least one arrival, and of which r time units are more than K time units

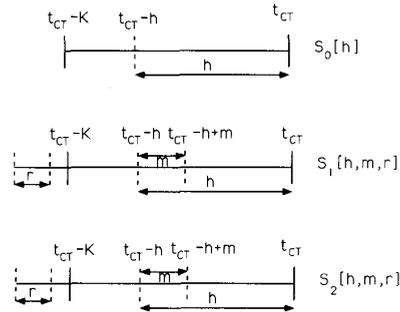


Fig. 1. Definition of states.

in the past, $0 \leq h \leq K$, $0 \leq m \leq K - h$, $0 \leq r \leq K - h - m$.

$S_2[h, m, r]$: In this state there are no untransmitted arrivals in the time interval $(t_{CT} - K, t_{CT} - h)$. The time interval $(t_{CT} - h, t_{CT} - h + m)$ is part of an interval of length $(m + r)$ in which we know that there are at least two arrivals, and of which r time units are more than K time units in the past, $0 \leq h \leq K$, $0 \leq m \leq K - h$, $0 \leq r \leq K - h - m$.

The value iteration equations corresponding to these three sets of states are listed in the Appendix. The reward is set to 1 for a successful transmission and 0 for a collision or an idle slot. As a result, the maximal gain per transition corresponds to the throughput. $VS_0^n[h]$, $VS_1^n[h, m, r]$, and $VS_2^n[h, m, r]$ are the expected rewards corresponding to states $S_0[h]$, $S_1[h, m, r]$, and $S_2[h, m, r]$, respectively, in the next n steps if the optimal policy is followed. The event of observing an idle, success or collision if an interval of length j is enabled is represented by $q_j = 0$, $q_j = 1$ or $q_j \geq 2$, respectively. Note that the three classes of states $S_0[h]$, $S_1[h, 0, r]$, and $S_2[h, 0, r]$ are identical.

The CRA determined by (A1)–(A3) is a nonnested CRA, unlike, for example [1]. This is due to the fact that we allow enabled sets that are not subsets of busy sets for states $S_1[h, m, r]$ and $S_2[h, m, r]$ when $m \leq h - (K - 1)$ and $K - 1 < h \leq K$. For this set of states, it is allowable to enable a part of the interval of length $h - (K - 1) - m$, where the number of arrivals has the Poisson distribution, along with the interval of length m corresponding to the part of the busy interval with packets that have not exceeded their deadlines. Since this interval will be lost by the next slot and lead to a transition to the $S_0[K]$ state in any case, this does not introduce new states. This option was introduced because it may lead to a higher probability of success. If this option is not allowed in (A2) and (A3), they would correspond to a nested CRA as described in [7].

To make the state space finite, the time axis is discretized by dividing each time unit into sufficiently small "minislots." The number of minislots per slot time was chosen by increasing their number until there was no appreciable change in the loss. Losses were computed for both the nested and nonnested CRA's. Because the state space increases rapidly with K , the performance of these CRA's could be computed only for

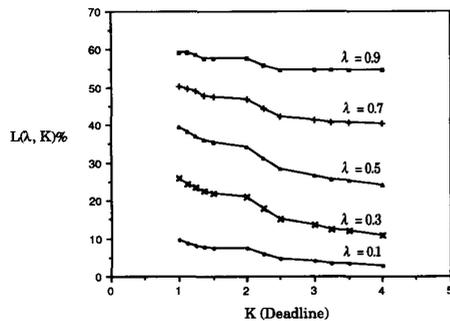


Fig. 2. Loss versus deadline for nonnested CRA.

values of $K \leq 4$. The loss for the nonnested CRA is plotted as a function of K for a range of values of λ in Fig. 2. As expected, the losses decrease with K for a fixed value of λ . The slope of the loss curve showed an interesting discontinuity for integer values of K . The nonnested variation of the CRA described earlier had lower losses than the nested CRA for values of $1 < K \leq 2$ (see Table I). The performance of the nested and nonnested CRA's is identical for values of K outside this range. We believe, however, that other nonnested CRA's can lead to an additional reduction in the loss.

IV. CONCLUSIONS

We considered the problem of devising collision resolution algorithms (CRA's) for a time-constrained multiple-access channel with ternary feedback. Poisson arrivals and a fixed deadline K for all packets were assumed. In [4], an efficient numerical technique allowed the computation of losses for a CRA for large values of K ($K \leq 30$). In contrast, we searched for the optimal CRA within a class of first-come, first-served (FCFS) window CRA's for the case when K is small ($K \leq 4$). The value iteration method was used to determine the optimal enabled time intervals for each state and the packet losses corresponding to different packet arrival rates and time

TABLE I
COMPARISON OF NESTED AND NONNESTED CRA'S FOR $1 < K \leq 2$

λ	K	Loss (%)	
		Nested CRA	Nonnested CRA
0.1	1.125	8.86	8.66
	1.25	8.09	8.04
	1.375	7.62	7.62
	1.5	7.44	7.44
	2.0	7.25	7.25
0.3	1.125	25.67	24.46
	1.25	23.62	23.34
	1.375	22.38	22.37
	1.5	21.89	21.85
	2.0	20.97	20.97
0.5	1.125	39.35	38.11
	1.25	37.56	37.14
	1.375	35.97	35.95
	1.5	35.42	35.41
	2.0	34.28	34.28
0.7	1.125	50.34	49.62
	1.25	49.44	48.94
	1.375	47.86	47.84
	1.5	47.49	47.47
	2.0	46.78	46.78
0.9	1.125	59.34	59.13
	1.25	59.18	58.66
	1.375	57.84	57.70
	1.5	57.74	57.71
	2.0	57.66	57.66

constraints. In addition, a nonnested CRA was shown to lead to lower losses for a range of deadline values. We believe that the technique used herein can be applied to the design of *adaptive* FCFS window CRA's for larger values of K , since it indicates how the window parameters should be varied for states with values of h close to K , that is, for those states where packets in the sliding window are close to their deadlines.

APPENDIX

$$VS_0^{n+1}[h] = \begin{cases} \max_{0 < j \leq h} \{ P(q_j = 0)VS_0^n[h - j + 1] + P(q_j = 1)\{VS_0^n[h - j + 1] + 1\} + P(q_j \geq 2)VS_2^n[h + 1, j, 0] \}, \\ \text{for } h \leq K - 1 \\ \max \left[\begin{aligned} & \max_{0 < j \leq h - (K - 1)} \{ P(q_j = 1) + VS_0^n[K] \}, \\ & \max_{h - (K - 1) < j \leq h} \{ P(q_j = 0)VS_0^n[h - j + 1] + P(q_j = 1)\{VS_0^n[h - j + 1] + 1\} \\ & \quad + P(q_j \geq 2)VS_2^n[K, K - 1 - h + j, h - K + 1] \} \} \end{aligned} \right], \\ \text{for } K - 1 < h \leq K. \end{cases} \tag{A1}$$

