

Analyzing a Full-Duplex Cellular System

Sanjay Goyal, Pei Liu, Sha Hua, and Shivendra Panwar
Polytechnic Institute of New York University
{sgoyal01, shua01}@students.poly.edu, pliu@poly.edu, panwar@catt.poly.edu

Abstract—Recent progress in single channel full-duplex (SC-FD) radio design [1]–[4] has attracted the attention of many researchers. A SC-FD transceiver is capable of transmitting and receiving on the same frequency at the same time, which will have a great impact on the design and performance of current wireless networks that are based on half duplex designs. This paper analyzes the effects of adopting SC-FD enabled base stations in a cellular system with legacy mobile stations. We use a multi-cell analytical model based on stochastic geometry to derive the theoretical performance gain of such a system. To validate the performance using a realistic setting, we conduct extensive simulations for a multi-cell OFDMA system. Both sets of results show that a full-duplex design for a cellular system, while not quite doubling system capacity, does greatly increase capacity over traditional cellular systems. Our results show that the uplink, compared with the downlink, is more susceptible to the extra interference caused by using the same frequency in both directions.

I. INTRODUCTION

Traditional radio transceivers are generally not able to receive and transmit on the same frequency band because of the crosstalk between the transmitter (Tx) and the receiver (Rx) circuits. Given that the intended received signal over the air is one million times or more weaker than the transmitted signal, it is very difficult, if not impossible, to detect the received signal under internal interference from the transmission circuits/antennas. Thus, most of today’s bidirectional systems rely on orthogonal signaling dimensions, such as time (TDD) or frequency (FDD) division duplexing, to be able to transmit and receive at the same time.

Full duplex communication has the potential to double the capacity through the removal of a separate frequency band/time slot for both forward and reverse links, and there has been much recent interest in full-duplex communication systems operating on a common carrier. Advances in antenna designs and analog/digital interference cancellation circuits have shown significant progress in canceling the self-interference, and make the design of a single channel full duplex (SC-FD) communication system a possibility [1]–[4]. Choi et al. [1] designed the first practical single channel wireless full-duplex system. They also presented full-duplex radio design [2] with a different feed-forward technique using a Balun and two separate antennas. Duarte et al. in [3] presented a SC-FD design using antenna separation, analog and digital cancellation. Knox [4] presented a SC-FD system using only a single antenna.

In this paper, we study the effects of deploying SC-FD enabled base stations (BSs) in a cellular network with legacy mobile stations (MSs). We assume perfect cancellation of the crosstalk between Tx and Rx circuits at BSs. While this is

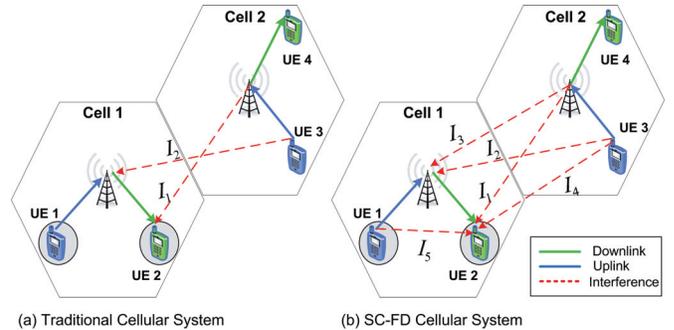


Fig. 1. An exemplary scenario for Traditional versus SC-FD cellular system far from true today, sufficient progress is being made in this direction for us to start considering this model and its implications. In a SC-FD cellular system, a BS is able to schedule an uplink and a downlink user on the same frequency at the same time, which is not possible in a traditional system. At first glance, scheduling an uplink and a downlink user with the same resources should simply double the capacity/throughput of the traditional system. However, as we discovered in this work, extra inter-cell and intra-cell interference caused by using the same channel in both directions limits the gain that can be reaped from full duplex designs.

The impact of deploying SC-FD enabled BSs in a multi-cell scenario is shown in Figure 1. For simplicity, we consider only two neighboring hexagonal cells. We analyze the impact on users of cell 1; the same analysis can be directly applied to users of cell 2. As shown in Figure 1(a), in a traditional cellular system, a downlink user (UE 2) only gets interference from a neighboring BS operating on the same downlink channel (i.e. I_1 from BS of cell 2), but in the case of a SC-FD cellular system, as shown in Figure 1(b), it also gets interference from uplink users operating on the same channel in both cells (i.e. I_4 from UE 3 of cell 2, and I_5 from UE 1 in its own cell). This is because, in the SC-FD case, both BSs also allocate the same downlink channel to their uplink users. Similarly, consider the case of an uplink user (UE 1). In a traditional cellular system, it only gets interference from a neighboring cell’s uplink user operating on the same channel (i.e. I_2 from UE 3 of cell 2), but in the SC-FD case, this user also gets interference from a neighboring BS operating on the same channel in downlink (i.e. I_3 from BS of cell 2). In such a SC-FD implementation, both uplink and downlink users experience higher co-channel interference from both neighboring cells and the cell it belongs to, which does not occur in legacy designs. Thus, a SC-FD cellular system does not simply double the capacity, since additional interference limits the potential capacity gain achievable by the SC-FD feature.

There only exist a few papers analyzing the SC-FD enabled system performance, and all are in different contexts from ours. In [5], Fang et al. consider the SC-FD enabled ad-hoc wireless network. They study the cross layer optimization problem for choosing routes to maximize the total profit of multiple users, and to minimize the network power consumption for such a network. In [6], Singh et al. proposed a novel MAC called ContraFlow that exploits the benefits of SC-FD implementation in wireless LAN (i.e. IEEE 802.11). They use the SC-FD feature to eliminate the hidden terminal problem and to improve the fairness and efficiency of the network. In [7], Cheng et al. analyzed the performance of a SC-FD enabled cognitive radio network (CRN). They compared the primary user's packet loss rate in a SC-FD enabled CRN with a traditional CRN. The same authors also analyzed a QoS based power allocation for a point to point SC-FD enabled wireless link in [8]. There are some papers in the literature like [9], and the references therein, which analyzed the performance of SC-FD enabled relay networks. Barghi et al. [10] examined the throughput gain of a SC-FD cellular system, but with only single cell. The single cell scenario does not consider the new inter-cell interference introduced in both the uplink and downlink channels. As we demonstrate, inter-cell interference generated due to the SC-FD feature is a major factor contributing to a decrease in the performance gain of a SC-FD system.

In this paper, we focus on studying and improving the performance of a SC-FD cellular system in comparison to traditional cellular systems. This paper provides a multi-cell analytical model to derive both downlink and uplink throughput for a SC-FD cellular system using stochastic geometry. We also address the problem of resource allocation for a SC-FD enabled OFDMA based multi-cell cellular system. Finally we compare the performance of a SC-FD cellular system with a traditional cellular system by generating the results from both an analytical model, as well as multi-cell OFDMA simulations.

The remainder of the paper is organized as follows. Section II presents an analytical model for a SC-FD cellular system to derive network capacity theoretically. In Section III, an OFDMA based multi-cell resource allocation scheme for a SC-FD cellular system is studied to determine the capacity for a practical cellular system. The performance evaluation is presented in Section IV. Finally, Section V concludes the paper.

II. ANALYTICAL MODELING

Traditionally, most researchers model cellular systems by assuming the placement of BSs according to a regular geometry (e.g. hexagonal, grid). However, these highly idealized models are far from accurate in terms of modeling interference and capacity. Recently, a general model based on stochastic geometry was proposed [11] [12], in which the BS and MS locations are assumed to be random according to a certain probability distribution. To derive the capacity for a SC-FD cellular system, our analysis follows the methods presented in [11] and [12], and derives the per channel average rate for

both uplink and downlink users for that system.

We assume the location of BSs follows a homogenous Poisson point process (PPP) Φ with density λ . For simplicity, we also assume that uplink users are distributed according to an independent homogenous PPP Ω with the same density λ . We assume that a user is connected to its closest BS. Moreover, for the uplink direction we assume that each BS has one active uplink user scheduled. We derive the per channel downlink and uplink average rate separately in the following sections.

A. Average Downlink Rate

In this section, we analyze the downlink performance in a SC-FD system. Without losing generality, we assume a typical downlink user d_0 is located at the origin. If it is associated to the BS b_0 , its SINR can be expressed as:

$$SINR_d = \frac{P_b h r^{-\alpha_1}}{N_0 W_c + I_{bs} + I_{up}}, \quad (1)$$

where I_{bs} and I_{up} is the total interference received at the downlink user d_0 from all other BSs, and all uplink users, respectively.

$$I_{bs} = P_b \sum_{b \in \Phi \setminus b_0} g_b R_b^{-\alpha_1}, \quad I_{up} = P_m \sum_{u \in \Omega} k_u D_u^{-\alpha_2}, \quad (2)$$

where W_c denotes the bandwidth used per channel. P_b and P_m are the transmission power of BS and MS on a channel, respectively. The distances of the downlink user d_0 from its BS b_0 , interfering BS b and interfering uplink user u are given by r , R_b , D_u , respectively. The path loss exponents α_1 and α_2 are used for the channel between the BS and MS, and for the channel between a pair of MS, respectively. N_0 is the noise power density. All channels are considered with Rayleigh fading (with $2\sigma^2 = 1$). Here, h , g_b , k_u are the channel fading for the channels of downlink user d_0 with its BS b_0 , with another interfering BS b and with an uplink user u , respectively.

Theorem 1: The average rate of a downlink user on a channel is given by:

$$\begin{aligned} \bar{R}_d(\lambda, \alpha_1, \alpha_2) &= W_c \mathbb{E}[\ln(1 + SINR_d)] = W_c \cdot \\ &\int_{r>0} 2\pi\lambda r e^{-\lambda\pi r^2} \int_{t>0} e^{-\frac{r^{\alpha_1}(e^t-1)N_0W_c}{P_b}} F_1(r, t) F_2(r, t) dt dr, \end{aligned} \quad (3)$$

where,

$$\begin{aligned} F_1(r, t) &= \exp\left(-\pi\lambda r^2 (e^t - 1)^{\frac{2}{\alpha_1}} \int_{(e^t-1)^{-\frac{2}{\alpha_1}}}^{\infty} \frac{1}{(1+x)^{\frac{\alpha_1}{2}}} dx\right), \\ F_2(r, t) &= \exp\left(-\pi\lambda r^{\left(\frac{2\alpha_1}{\alpha_2}\right)} \left(\frac{P_m(e^t-1)}{P_b}\right)^{\frac{2}{\alpha_2}} \int_{y>0} \frac{1}{(1+y)^{\frac{\alpha_2}{2}}} dy\right). \end{aligned}$$

Proof: This proof follows and extends the derivation given in [11] for a traditional downlink user. Each downlink user connects to its closest BS. We use r to denote the distance between the user and its associated BS. The cumulative density function (CDF) of the distance r is then:

$$\begin{aligned} F_r(R) &= Pr(r \leq R) = 1 - Pr(r > R) \\ &= 1 - Pr(\text{no BS closer than } R) = 1 - e^{-\lambda\pi R^2}. \end{aligned} \quad (4)$$

Thus the probability density function (pdf) is :

$$f_r(r) = \frac{dF_r(r)}{dr} = e^{-\lambda\pi r^2} 2\pi\lambda r. \quad (5)$$

The average downlink rate per hertz is:

$$\begin{aligned} & \mathbb{E}[\ln(1 + SINR_d)] \\ &= \int_{r>0} \int_{t>0} f_r(r) P[\ln(1 + SINR_d) > t] dt dr \\ &= \int_{r>0} e^{-\lambda\pi r^2} \int_{t>0} P\left[h > \frac{(e^t - 1)r^{\alpha_1}(N_0W_c + I_{bs} + I_{up})}{P_b}\right] dt 2\pi\lambda r dr \\ &\stackrel{(a)}{=} \int_{r>0} e^{-\lambda\pi r^2} \int_{t>0} \mathbb{E}\left[\exp\left(-\frac{(e^t - 1)r^{\alpha_1}(N_0W_c + I_{bs} + I_{up})}{P_b}\right)\right] dt 2\pi\lambda r dr \\ &= \int_{r>0} 2\pi\lambda r e^{-\lambda\pi r^2} \int_{t>0} e^{-\frac{r^{\alpha_1}(e^t - 1)N_0W_c}{P_b}} F(r, t) dt dr, \quad (6) \end{aligned}$$

where,

$$\begin{aligned} F(r, t) &= \mathbb{E}\left[\exp\left(-\frac{(e^t - 1)r^{\alpha_1}(I_{bs} + I_{up})}{P_b}\right)\right] \\ &\stackrel{(b)}{=} \mathbb{E}\left[e^{-sP_b \sum_{b \in \Phi \setminus b_0} g_b R_b^{-\alpha_1}} e^{-sP_m \sum_{u \in \Omega} k_u D_u^{-\alpha_2}}\right] \\ &= \mathbb{E}\left[\prod_{b \in \Phi \setminus b_0} e^{-sP_b g_b R_b^{-\alpha_1}} \prod_{u \in \Omega} e^{-sP_m k_u D_u^{-\alpha_2}}\right] \\ &\stackrel{(c)}{=} \mathbb{E}\left[\prod_{b \in \Phi \setminus b_0} \mathbb{E}_{g_b}(e^{-sP_b g_b R_b^{-\alpha_1}}) \prod_{u \in \Omega} \mathbb{E}_{k_u}(e^{-sP_m k_u D_u^{-\alpha_2}})\right] \\ &\stackrel{(d)}{=} \mathbb{E}\left[\prod_{b \in \Phi \setminus b_0} \frac{1}{1 + sP_b R_b^{-\alpha_1}} \prod_{u \in \Omega} \frac{1}{1 + sP_m D_u^{-\alpha_2}}\right] \\ &\stackrel{(e)}{=} \mathbb{E}_{\Phi}\left[\prod_{b \in \Phi \setminus b_0} \frac{1}{1 + sP_b R_b^{-\alpha_1}}\right] \mathbb{E}_{\Omega}\left[\prod_{u \in \Omega} \frac{1}{1 + sP_m D_u^{-\alpha_2}}\right] \\ &= G_1(s, r) G_2(s, r). \quad (7) \end{aligned}$$

In above equations, (a) follows from the fact that $h \sim \exp(1)$; (b) follows by substituting the values of I_{bs} and I_{up} from Eq. (2) and replacing $(r^{\alpha_1}(e^t - 1)/P_b)$ by s ; (c) follows from the independence of g_b and k_u ; (d) follows from $g_b \sim \exp(1)$, and $k_u \sim \exp(1)$; (e) follows from the fact that the BS distribution and the uplink user distribution are independent. In Eq. (7),

$$G_1(s, r) = \exp\left(-2\pi\lambda \int_r^\infty \left(1 - \frac{1}{1 + sP_b v^{-\alpha_1}}\right) v dv\right), \quad (8)$$

which follows from the probability generating functional (PGFL) of the PPP [11] [12]. The integration limits are from r to ∞ since the closest interferer BS is at least at a distance r .

Putting the value of $s = (r^{\alpha_1}(e^t - 1)/P_b)$, and replacing $(v/r(e^t - 1)^{\frac{1}{\alpha_1}})^2$ with x , we get :

$$G_1(s, r) = e^{\left(-\pi r^2 \lambda (e^t - 1)^{\frac{2}{\alpha_1}} \int_{(e^t - 1)^{-\frac{2}{\alpha_1}}}^\infty \frac{1}{(1 + x^{\frac{\alpha_1}{2}})} dx\right)}. \quad (9)$$

Similarly,

$$G_2(s, r) = \exp\left(-2\pi\lambda \int_{r>0} \left(1 - \frac{1}{1 + sP_m z^{-\alpha_2}}\right) z dz\right), \quad (10)$$

where the integration limits are from 0 to ∞ since the closest interferer uplink user can be at any distance. After plugging

in the value of s and then replacing $\left(\frac{z}{r^{\frac{\alpha_1}{2}} \left(\frac{P_m(e^t - 1)}{P_b}\right)^{\frac{1}{\alpha_2}}}\right)^2$ to x , we get :

$$G_2(s, r) = e^{\left(-\pi r^{\left(\frac{2\alpha_1}{\alpha_2}\right)} \lambda (P_m(e^t - 1)/P_b)^{\frac{2}{\alpha_2}} \int_{y>0} \frac{1}{(1 + y^{\frac{\alpha_2}{2}})} dy\right)}. \quad (11)$$

Combining the equations Eqs. (9) and (11) with Eq. (7), and then with Eq. (6), we obtain the result.

B. Average Uplink Rate

In this section, we derive the uplink performance in a SC-FD system. Assuming a typical uplink user u_0 is associated with BS b_0 , which is located at the origin. The SINR for this uplink user can be expressed as:

$$SINR_u = \frac{P_m h' r^{-\alpha_1}}{N_0 W_c + I'_{up} + I'_{bs}}, \quad (12)$$

where I'_{bs} is the total interference received at BS (for the uplink user u_0) from all other BSs that are serving their own downlink user due to SC-FD transmission, and I'_{up} is the interference from other uplink users in its neighboring cells.

$$I'_{up} = P_m \sum_{u \in \Omega \setminus u_0} k'_u X_u^{-\alpha_1}, \quad I'_{bs} = P_b \sum_{b \in \Phi \setminus b_0} g'_b B_b^{-\alpha_3}, \quad (13)$$

where r is the distance between uplink user u_0 and its BS b_0 . The distances of BS b_0 from interfering uplink user u and interfering BS b are given by X_u and B_b , respectively. The path loss exponent α_3 is used for the channel between two BSs. The channel fading between user u_0 and BS b_0 , an other interfering BS b and BS b_0 , and interfering MS u to BS b_0 are denoted by h' , g'_b , and k'_u , respectively. All channels are subjected to Rayleigh fading (with $2\sigma^2 = 1$).

Theorem 2: The average rate of an uplink user on a channel is given by:

$$\begin{aligned} \bar{R}_u(\lambda, \alpha_1, \alpha_3) &= W_c \mathbb{E}[\ln(1 + SINR_u)] = W_c. \\ &= \int_{r>0} 2\pi\lambda r e^{-\lambda\pi r^2} \int_{t>0} e^{-\frac{r^{\alpha_1}(e^t - 1)N_0W_c}{P_m}} F'_1(r, t) F'_2(r, t) dt dr, \quad (14) \end{aligned}$$

where,

$$\begin{aligned} F'_1(r, t) &= \exp\left(-\pi\lambda r^2 (e^t - 1)^{\frac{2}{\alpha_1}} \int_{(e^t - 1)^{-\frac{2}{\alpha_1}}}^\infty \frac{1}{(1 + x^{\frac{\alpha_1}{2}})} dx\right), \\ F'_2(r, t) &= \exp\left(-\pi\lambda r^{\left(\frac{2\alpha_1}{\alpha_3}\right)} \left(\frac{P_b(e^t - 1)}{P_m}\right)^{\frac{2}{\alpha_3}} \int_{y>0} \frac{1}{(1 + y^{\frac{\alpha_3}{2}})} dy\right). \end{aligned}$$

Proof: In this case, we also consider that an uplink user is associated to its closest BS. Thus, the PDF of the distance between the uplink user and its BS is the same as given in Eq. (5). For an uplink user (u_0), all other BSs and uplink users located at a distance larger than r (where r is the distance of (u_0) from its closest BS (b_0)) will generate interference for it at its BS (b_0). For further details of the proof, one can refer to the derivation given for *Theorem 1*; it follows the same steps given there.

Average downlink and uplink rates in traditional cellular systems can be found in [11] and [12], respectively. The derived rates for both SC-FD and traditional case are computed numerically for performance evaluation in this paper.

III. AN OFDMA-BASED MULTI-CELL SYSTEM

In the previous section, we considered a system with random Poisson BS deployment. However, our analysis simplifies the real system in several aspects: 1) Most modern cellular systems are OFDMA based, where there are multiple non-overlapping narrow band subchannels; 2) It has a fixed allocation of one channel to one user, whereas in an OFDMA based system, subchannels are allocated to each user according to its demand, channel state and fairness among all users. In this section, we consider an OFDMA based multi-cell network to study how SC-FD enabled BSs improve the capacity of cellular systems. We also provide an efficient sub-optimal resource allocation algorithm in a SC-FD OFDMA system.

In an OFDMA system, total bandwidth is divided into several orthogonal subchannels (i.e. a group of sub-carriers) and, one frame is divided into several time slots. The combination of a subchannel and a time slot is called a *resource block (RB)*, which is the minimum unit for resource allocation. In a half-duplex network, each BS can assign a RB to only one user to avoid intra-cell interference. RBs for uplink and downlink are different, so downlink transmission does not create interference in the uplink direction and vice versa. However, in a SC-FD cellular system, in which we assume perfect cancelation of crosstalk interference, a RB can be assigned to one uplink and one downlink transmission at the same time. As described in Section I, this will create more intra-cell and inter-cell interference for both uplink and downlink users. In this work, we consider two different resource allocation algorithms. The first is a simple round-robin scheduling algorithm, which does not consider any instantaneous channel gains and interference information availability at the scheduler. The scheduler allocates its resources sequentially to all users, so that each user has an approximately equal number of resource blocks allocated. Second, we consider a resource allocation scheme which is based on proportional fairness, and considers that the channel state information for all channels is available at a Radio Network Controller (RNC), which coordinates all the BSs. For this algorithm, we give the problem formulation and scheduling algorithm for a SC-FD cellular system.

A. Problem Formulation

We consider the set of all BSs in the network as Φ , while U_b and D_b are the set of uplink and downlink users belonging to a BS b . There are a total of C sub-channels that can be used for both downlink and uplink at the same time. The goal of the scheduler is to allocate channels to maximize the long-term sum rate to/from all mobile stations, subject to some fairness constraint. According to [13], such an objective is equivalent to maximizing the aggregate marginal utility in each time slot. Based on that, the problem can be formulated as shown in (15). In (15), $X(u, d, b, c)$ indicates the assignment of sub-channel c to uplink user u and downlink user d of base station b , and the weight function $W(u, b, d, c)$ represents the resulting net marginal utility. This utility function takes interference into account. The constraint limits the assignment of a sub-channel

to at most one user in each direction for each BS.

$$\max_{X(u,d,b,c)} : \sum_{c=1}^C \sum_{b \in \Phi} \sum_{u \in U'_b} \sum_{d \in D'_b} W(u, d, b, c) X(u, d, b, c),$$

subject to:

$$\sum_{u \in U'_b} \sum_{d \in D'_b} X(u, d, b, c) \leq 1, \forall b \in \Phi; \forall c \in [1, C], \quad (15)$$

$$X(u, d, b, c) \in \{0, 1\},$$

$$\forall u \in U'_b, \forall d \in D'_b, \forall b \in \Phi, \forall c \in [1, C],$$

$$U'_b = U_b \cup \phi, \quad D'_b = D_b \cup \phi.$$

The weight function is:

$$W(u, d, b, c) = \frac{r_{u,b,c}}{\bar{r}_{u,b}} + \frac{r_{d,b,c}}{\bar{r}_{d,b}}, \forall u \in U'_b, \forall d \in D'_b, \quad (16)$$

which represents the sum of the marginal utility gained by scheduling uplink user u and a downlink user d on channel c . This definition of marginal utility gain guarantees proportional fairness [13]. Here, $r_{u,b,c}$ and $r_{d,b,c}$ denote the instantaneous rate for uplink and downlink, respectively, which can be calculated as:

$$\begin{aligned} r_{u,b,c} &= W_c \log_2(1 + SINR_{u,b,c}), \\ r_{d,b,c} &= W_c \log_2(1 + SINR_{d,b,c}), \end{aligned} \quad (17)$$

where $SINR_{u,b,c}$ is the SINR of uplink user u for channel c and BS b . It can be defined in a manner similar to Eq. (12). Here, only those other BS and uplink users, which are operating on channel c at the same time instant will create interference. Similarly, we can define $SINR_{d,b,c}$ for downlink user d with Eq. (1). Moreover in Eq. (16), $\bar{r}_{u,b}$ and $\bar{r}_{d,b}$ are the moving averages of the throughput of uplink user u and downlink user d of BS b . In each time slot t , they are updated as:

$$\begin{aligned} \bar{r}_{u,b}(t) &= \beta \bar{r}_{u,b}(t-1) + (1-\beta) r_{u,b,c}, \\ \bar{r}_{d,b}(t) &= \beta \bar{r}_{d,b}(t-1) + (1-\beta) r_{d,b,c}, \end{aligned} \quad (18)$$

where β is a weight constant taking a value between 0 and 1.

B. Scheduling Algorithm

In a multi-cell scenario, due to the complexity of the optimal resource allocation problem, only heuristic methods exist so far for traditional cellular systems [14]–[17]. Motivated by them, for our SC-FD system, we provide a centralized greedy algorithm to achieve a sub-optimal solution. The algorithm is shown in Algorithm 1. It first initializes the matrices that contain the allocation results (Line 1). \mathbf{N} is a boolean matrix, for which entry $N_{b,c,t} = 1$ means that BS b uses subchannel c for the downlink in slot t . Matrices \mathbf{H} and \mathbf{G} contain the information of scheduled uplink users and downlink users respectively. The entry $H_{b,c,t}$ in \mathbf{H} contains the index of scheduled uplink user of BS b on subchannel c in slot t , if any, otherwise it will be zero. Similarly, entry $G_{b,c,t}$ of matrix \mathbf{G} contains the index of the scheduled downlink user (if any, otherwise 0) of BS b on subchannel c in slot t . The subchannels are allocated sequentially (Line 3). For each subchannel, the

Algorithm 1: SC-FD Multi-Cell Resource Allocation Algorithm

```
1  $\mathbf{N}_{B \times N \times T} = 0$ ;  $\mathbf{H}_{B \times N \times T} = 0$ ;  $\mathbf{G}_{B \times N \times T} = 0$ ;  
2 for  $t = 1$  to  $T$  do  
3   for  $c = 1$  to  $C$  do  
4     while  $\Phi \neq \emptyset$  do  
5        $b \leftarrow$  BS with the least number of assigned  
        channels in  $\Phi$ ;  
6        $\Phi \leftarrow \Phi \setminus b$ ;  
7        $U'_b = U_b \cup \{0\}$ ;  $D'_b = D_b \cup \{0\}$ ;  
8        $\{u^*, d^*\} =$   
         $\arg \max_{u \in U'_b, d \in D'_b} \{Get\_Utility(b, c, t, u, d)\}$ ;  
9        $\Delta U_{max} = Get\_Utility(b, c, t, u^*, d^*)$ ;  
10      if  $\Delta U_{max} > 0$  then  
11         $\mathbf{H} \leftarrow$  set  $\mathbf{H}_{b,c,t} = u^*$  in  $\mathbf{H}$ ;  
12         $\mathbf{G} \leftarrow$  set  $\mathbf{G}_{b,c,t} = d^*$  in  $\mathbf{G}$ ;  
13        if  $d^* \neq 0$  then  
14           $\mathbf{N} \leftarrow$  set  $\mathbf{N}_{b,c,t} = 1$  in  $\mathbf{N}$ ;
```

BSs with a fewer number of assigned subchannels are given higher priority to maintain fairness (Line 5). Then for each chosen BS, we find the user(s) with the maximum utility gain ΔU on the corresponding subchannel (Line 8) using a function $Get_Utility(\cdot)$. Finally, the algorithm chooses the user(s) with the largest utility gain ΔU_{max} for the given BS and subchannel. If ΔU_{max} is positive, it schedules the chosen user(s) and updates the matrices accordingly (Lines 10-14).

Next, let us see how the function $Get_Utility(\cdot)$ works. As shown in Algorithm 2, it calculates the utility gain ΔU , which is the difference between the gain in the marginal utility of the chosen user(s) ($U_{gain,u}$ or $U_{gain,d}$ or both) and loss in marginal utility of other uplink and downlink users ($|U_{loss_uplink}|$ and $|U_{loss_downlink}|$) on the same subchannel due to new interference generated from the chosen user(s) (Line 8). Here $U_{gain,i}$ (e.g. $U_{gain,u}$, $U_{gain,d}$), which is the gain in utility due to scheduling of user i (say for BS b , subchannel c and slot t) is given in Line 5. Total utility loss for other uplink users and downlink users, U_{loss_uplink} and $U_{loss_downlink}$, are given in Line 6 and 7 respectively. In all the above equations, the function $U(r_{i,k,c}(\mathbf{N}, \mathbf{H})) = \frac{r_{i,k,c}}{\bar{r}_{i,k}}$ is the same as the marginal utility gain of a user defined in Section III-A.

IV. PERFORMANCE EVALUATION

In this section, we evaluate the performance of a SC-FD cellular system under three scenarios. In the first scenario, we numerically calculate the uplink and downlink throughput, following the analytical model in Section II. In the second scenario, we simulate an OFDMA based multi-cell network with a hexagonal grid layout. The channels are allocated to MSs according to the round-robin scheduling algorithm described in Section III. In the third scenario, we maintain the same settings as in scenario 2 and employ the scheduling algorithm proposed in Section III-B. In each scenario, to simulate the traditional system, we consider a TDD system, in which the downlink and uplink scheduling are performed independently in alternate time slots. We consider a total

Algorithm 2: Function $Get_Utility(b, c, t, u, d)$

```
1  $\mathbf{H}' \leftarrow$  set  $\mathbf{H}_{b,c,t} = u$  in  $\mathbf{H}$ ;  
2  $\mathbf{G}' \leftarrow$  set  $\mathbf{G}_{b,c,t} = d$  in  $\mathbf{G}$ ;  
3 if  $d \neq 0$  then  
4    $\mathbf{N}' \leftarrow$  set  $\mathbf{N}_{b,c,t} = 1$  in  $\mathbf{N}$ ;  
5  $U_{gain,i} \leftarrow U(r_{i,b,c}(\mathbf{N}', \mathbf{H}'))$ ,  $i = u, d$ ;  
6  $U_{loss\_uplink} \leftarrow$   
    $\sum_{i=H_{k,c,t}:i \neq 0, \forall k \in \Phi \setminus b} \{U(r_{i,k,c}(\mathbf{N}', \mathbf{H}')) - U(r_{i,k,c}(\mathbf{N}, \mathbf{H}))\}$ ;  
7  $U_{loss\_downlink} \leftarrow$   
    $\sum_{i=G_{k,c,t}:i \neq 0, \forall k \in \Phi \setminus b} \{U(r_{i,k,c}(\mathbf{N}', \mathbf{H}')) - U(r_{i,k,c}(\mathbf{N}, \mathbf{H}))\}$ ;  
8  $\Delta U = U_{gain,u} + U_{gain,d} - |U_{loss\_uplink}| - |U_{loss\_downlink}|$ ;  
9 return  $\Delta U$ ;
```

bandwidth of 20 MHz with 2048 sub-carriers divided into 64 subchannels. Total transmission power for both BS and MS is set to 23 dBm, which is split equally across all subchannels. A noise power spectrum density of -174 dBm/Hz is considered [18]. We model the channel with Rayleigh fading ($2\sigma^2 = 1$). The path loss for all types of channels (i.e. BS \leftrightarrow MS, MS \leftrightarrow MS, and BS \leftrightarrow BS) is calculated based on the Hata model for urban areas [19]. We considered a BS of height 32m and a MS of height 1.5m.

A. Numerical Evaluation of the Analytical Model

First, we numerically computed the results of the analytical model for the SC-FD system and the traditional cellular system given in Section II. We set the BS density, the average number of BSs in a unit area as $\lambda = 33 \cdot 10^{-8} m^{-2}$. Figure 2(a) shows the aggregate per cell throughput in uplink and downlink for both systems. It shows a capacity gain of 11% in the uplink and 91% in the downlink as compared with the traditional system. Thus, the SC-FD system increases the overall capacity of the traditional system significantly. However, the high inter-cell interference from neighboring BSs for an uplink user in a SC-FD system limits the performance gain in the uplink. This is because the BSs are usually deployed on the top of towers or building rooftops, which results in reduced path loss, and thus higher interference between neighboring BSs. However, in the case of the downlink, the large path loss between low height MSs results in less interference between them, which allows higher gain.

B. OFDMA-based Multi-cell Simulations

Second, we conducted the OFDMA-based simulations in a hexagonal grid layout. There are 7 neighboring hexagonal cells, each with a 1 km radius. 50 uplink and 50 downlink users are distributed uniformly in each cell. First, we generate the results for the simple round robin algorithm described in Section III. We calculate the aggregate per cell throughput for both the SC-FD system and the traditional cellular system. As shown in Figure 2(b), a gain of 57% in the uplink and 86% in the downlink can be achieved for a SC-FD system.

Then we applied our proposed scheduling algorithm as described in Section III-B to the hexagonal layout. As shown in Figure 2(c), there is a gain of 57% and 99% in aggregate per cell uplink and downlink throughput for the SC-FD system. However, this algorithm is more effective in increasing the SC-FD system gain over the round-robin algorithm in the

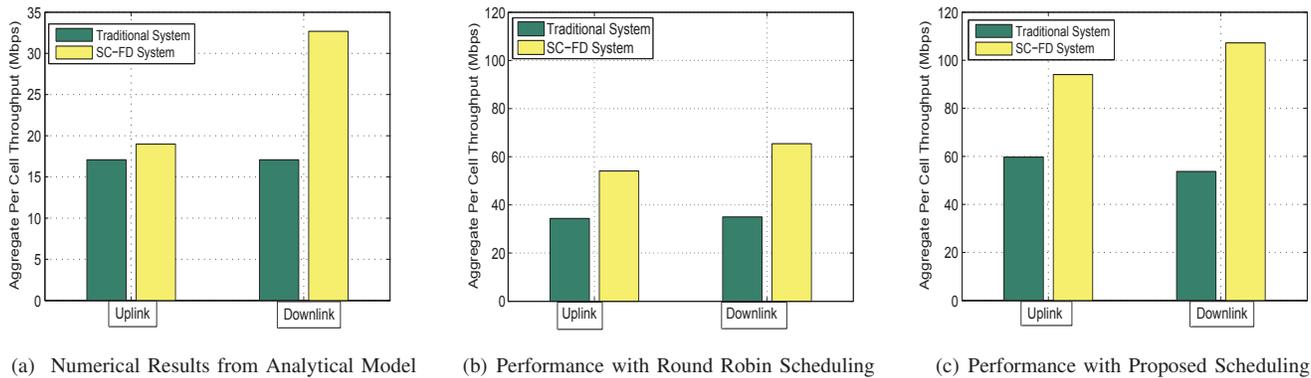


Fig. 2. Performance Comparison of SC-FD System with Traditional System

downlink direction (i.e. from **86%** to **99%**) as compared to the uplink direction. This is because, in the uplink, the interference is measured at the BS with the total interference consisting of two parts. The first one is the interference from other BSs that are transmitting to users in their own cells in the downlink direction; such interference is constant as long as the Tx power is fixed. The other source of interference is from the uplink transmissions of neighboring cells, for which, as long as the distribution of the MSs are fixed, the average interference does not vary according to the resource allocation. For the downlink case, on the other hand, most of the interference is from an uplink user in the same cell, which our proposed algorithm mitigates by trying to schedule a pair far enough from each other.

As we can see in Figure 2(b) and 2(c), our proposed algorithm increases the aggregate throughput in each direction significantly compared with a system that employs round-robin scheduler. This is due to the fact that channel gains and interference information is available at the scheduler, thus a better allocation of a subchannel can be achieved. The aggregate throughput of our analytical model shown in Figure 2(a) do not match with an OFDMA-based multi-cell simulation. This difference is due to the completely different distribution of BSs and users in both models. However, in all scenarios, the results show that SC-FD performs better in downlink, while strong co-channel interference limits the performance of full duplex designs in the uplink.

V. CONCLUSION

In this work, we investigate the performance of a SC-FD based current cellular system, and provide a tractable analytical model to derive the average per channel rate of uplink and downlink users. A suboptimal resource allocation algorithm for an OFDMA based SC-FD multi-cell cellular network is also proposed. Extensive performance evaluations are conducted with a hexagonal grid distribution. All of our results show that an SC-FD implementation can increase the aggregate throughput of current cellular system significantly in both downlink and uplink. However, uplink gain is limited by the increased interference caused by using the same frequency in two directions. As an extension of this work, we plan to employ intelligent frequency planning, power control and sectorization to alleviate the strong co-channel interference caused by full duplex radios, and we expect a higher throughput gain.

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