

# An Adaptive Control Scheme for Stabilizing TCP

Yi Fan, Zhong-Ping Jiang and Shivendra S. Panwar  
Department of Electrical and Computer Engineering

Polytechnic University  
Six Metrotech Center  
Brooklyn, 11201, U.S.A

yfan01@utopia.poly.edu, zjiang@control.poly.edu, panwar@catt.poly.edu

**Abstract**—Following the observations of many researchers that the performance of various AQM (Active Queue Management) schemes is sensitive to parameter variations, we propose stabilizing controllers that adapt to unknown or slowly varying parameters such as load factor, round trip time and link capacity. Our adaptive controllers are designed using new advances in control techniques and achieve asymptotic convergence to a desired equilibrium point. The proposed state feedback controller is designed using feedback linearization and back-stepping technique. A filter based design is proposed for adaptive output feedback control.

**Keywords** TCP, Congestion control, Asymptotic regulation.

## I. INTRODUCTION

Internet congestion control with stability considerations has been addressed in a large body of work after Jacobson proposed the first congestion control scheme [9]. The existing approaches such as drop-tail and RED (random early detection) are based on engineering intuition. Although they have alleviated Internet congestion in certain situations, their shortcomings have also been noted. For example, drop-tail tends to congest link buffers and cause high packet loss [8]. It is difficult to tune RED parameters to tradeoff between stability and system responsiveness [16][6]. With the help of the differential equation model introduced in [15] and [11], tools from control theory have been applied for the synthesis of new controllers with improved performance. In [5], a state feedback design is proposed to stabilize the queue length of bottleneck link buffer based on a linearized plant model. Since the Internet is a large scale system, it is difficult to measure the full states locally. An output feedback design is more preferable as it requires only limited output information. Hollot and Chait in [8] proposed a static output feedback controller (“proportional marking”) to stabilize the TCP/AQM closed loop system. The equilibrium queue length is characterized by network parameters and is thus not settable. To achieve regulation to a desired equilibrium queue length, the authors of [7] then proposed a “PI” controller instead of the “P” controller. They achieved local asymptotic queue length regulation based on a linearized model. This leaves open the design of an output feedback controller to stabilize the nonlinear network model with a larger domain of stability.

Research supported by the National Science Foundation under grants ANI-0081527 and ECS-0093176, and the New York State Center for Advanced Technology in Telecommunications (CATT) at Polytechnic University.

Robustness of the congestion control algorithms w.r.t. parametric uncertainty has also been addressed in past literature. Two approaches are used. One approach follows from a robust control perspective. Hollot, Misra, Towsley, and Gong [6] used frequency domain tools to calculate the stability margin of the linearized model. Similar stability results are obtained using nonlinear analysis: Alpcan and Basar [1], Wen and Arcak [17], Fan and Arcak [4]. Essentially their approaches are using a low gain to preserve stability in the face of delay and disturbance. These schemes are based on worst case analysis and sometimes lead to performance degradation such as capacity under-utilization.

Another approach to handle unknown network conditions is to use adaptive control methods when the network parameters are relatively fixed. Floyd, Gummadi and Shenker in [3] proposed an intuitive modification of RED to provide robustness to a range of parameter settings and supported their design with simulations rather than theoretical analysis. Zhang and Hollot et al [18] proposed a self-tuning AQM scheme for linearized network model. Their method is an indirect adaptive control method. They used the estimates for network parameters to update the RED controller parameters by applying certainty equivalence principle. Since RED has built-in instability [16], it will be interesting to consider designing a new controller to achieve both stabilization and parameter adaptation.

Inspired by the above work, our interest is to design an output feedback controller to stabilize a network with an unknown nonlinear model. We propose direct adaptive controller designs with the help of recent advances in adaptive and nonlinear control. The major contribution of our work is that we achieve desired queue length regulation for an unknown nonlinear network model. The closed loop system adapts to node capacity  $C$ , load factor  $N$  and round-trip time  $\tau$ .

## II. DYNAMIC MODEL AND DESIGN OBJECTIVE

The results of our paper are based on the fluid model proposed by Kelly [11], Misra, Gong and Towsley et al [15]. The following plant model is considered:

$$\dot{q} = N \frac{W}{\tau} - C \quad (1)$$

$$\dot{W} = \frac{1}{\tau} - \frac{W^2 + 2}{2\tau} p \quad (2)$$

The bottleneck link buffer queue length  $q$  and end host window size  $W$  are taken as the state variables.  $q$  is the measured output. The plant is valid for  $q \in [0, q_{ma}]$ ,  $W \in [0, W_{ma}]$  where  $q_{ma}$  and  $W_{ma}$  denote the link buffer size and the maximum window size.  $N$  and  $\tau$  represent the number of users (Load factor), link capacity and the round trip time, all of which are assumed unknown here. Equation (1) models the bottleneck queue accumulation as the integration of the excess of the packets sending rate over link capacity. Equation (2) models both the additive increase and multiplicative decrease window size evolution of TCP. Packet dropping or marking probability  $p$  in (2) is the control input. This model considers homogeneous multiple TCP connections, a single bottleneck link and delay free feedback situation. Detailed justification of this model can be found in [11], [15] and [14].

Our objective is to design a stabilizing output feedback control  $p$  to achieve asymptotic stability of the desired equilibrium point  $W^*$ ,  $q^*$  when the buffer queue length is considered as the measured output.

### III. ADAPTIVE STABILIZING CONTROLLER DESIGN

In this section we consider two adaptive control designs. The first one is a state feedback design, relying on both window size and queue length measurements. Our method is different from [5] in that:

- 1) Nonlinear backstepping method is used instead of linear frequency domain methods.
- 2) Unknown network parameters can be handled.

Our second design is an output feedback design relying only on link buffer queue length measurement.

#### A. Adaptive controller design using state feedback

With the objective of stabilizing TCP and parameter adaptation, we first consider the transformation:

$$\begin{aligned} x_1 &= q - q^* \\ x_2 &= W \\ u &= 1 - \frac{W^2 + 2}{2} p \end{aligned}$$

Systems (1) and (2) have been transformed into a linear form using above transformation:

$$\begin{aligned} \tau \cdot \dot{x}_1 &= N(x_2 - W^*) \\ \tau \cdot \dot{x}_2 &= u \end{aligned}$$

Then consider the Lyapunov function candidate:

$$2V_1 = \frac{\tau}{N} x_1^2 + \frac{1}{\gamma_w} (\theta_w - W^*)^2,$$

where  $\theta_w$  is an estimator for the unknown equilibrium window size  $W^*$ .  $\gamma_w > 0$  is an adaption tuning parameter. The update law for  $\theta_w$  is chosen as

$$\dot{\theta}_w = -\gamma_w x_1 \quad (3)$$

with the coordinates change

$$\begin{aligned} z_1 &= x_1 \\ z_2 &= x_2 + k_1 z_1 - \theta_w \end{aligned} \quad x$$

where  $k_1 > 0$  is a control gain, the system is changed into:

$$\begin{aligned} \tau \cdot \dot{z}_1 &= N(z_2 - k_1 z_1 + \theta_w - W^*) \\ \tau \cdot \dot{z}_2 &= u + N \cdot k_1 (z_2 - k_1 z_1 + \theta_w) - k_1 \cdot C + \gamma_w \cdot \tau \cdot z_1 \\ \dot{V}_1 &= -k_1 z_1^2 + z_1 z_2. \end{aligned}$$

Now consider Lyapunov function candidates:

$$\begin{aligned} 2V_2 &= 2V_1 + \tau \cdot z_2^2 \\ 2V &= 2V_2 + \frac{(\theta_N - N)^2}{\gamma_N} + \frac{(\theta_{C\tau} - C)^2}{\gamma_{C\tau}} + \frac{(\theta_\tau - \tau)^2}{\gamma_\tau} \end{aligned}$$

where  $\theta_N$ ,  $\theta_{C\tau}$  and  $\theta_\tau$  are estimates for unknown parameters  $N$  and  $\tau$ . Their adaptation laws are chosen as

$$\begin{aligned} \dot{\theta}_N &= \gamma_N z_2 k_1 (z_2 - k_1 z_1 + \theta_w) \\ \dot{\theta}_{C\tau} &= -\gamma_{C\tau} k_1 z_2 \\ \dot{\theta}_\tau &= \gamma_\tau \gamma_w z_1 z_2 \end{aligned} \quad (4)$$

where  $\gamma_N$ ,  $\gamma_{C\tau}$  and  $\gamma_\tau$  are adaption tuning parameters. Choose

$$u = -k_2 z_2 - z_1 - k_1 \theta_N (z_2 - k_1 z_1 + \theta_w) - k_1 \theta_{C\tau} - \gamma_w \theta_\tau z_1$$

where  $k_2 > 0$  is the control gain. Therefore,

$$\begin{aligned} \dot{V} &= \dot{V}_1 + \tau \cdot 2\dot{z}_2 \\ &+ \frac{(\theta_N - N)}{\gamma_N} \dot{\theta}_N + \frac{(\theta_{C\tau} - C)}{\gamma_{C\tau}} \dot{\theta}_{C\tau} + \frac{(\theta_\tau - \tau)}{\gamma_\tau} \dot{\theta}_\tau \\ &= -k_1 z_1^2 + z_1 z_2 + z_2 u \\ &+ \frac{1}{\gamma_N} \theta_N \dot{\theta}_N + \frac{1}{\gamma_{C\tau}} \theta_{C\tau} \dot{\theta}_{C\tau} + \frac{1}{\gamma_\tau} \theta_\tau \dot{\theta}_\tau \\ &= -k_1 z_1^2 + z_1 z_2 + z_2 u + k_1 z_2 \theta_N (z_2 - k_1 z_1 + \theta_w) \\ &- k_1 \theta_{C\tau} z_2 + \gamma_w \theta_\tau z_1 z_2 \\ &= -k_1 z_1^2 - k_2 z_2^2 \leq 0 \end{aligned}$$

Using LaSalle's invariance principle [10], and through the Lyapunov function  $V$ , it is obvious that the control law  $u$ , together with the adaptation laws (3) and (4), asymptotically regulates link buffer queue size  $q$  and source window size  $W$  to their equilibrium values  $q^*$  and  $W^*$ . The actual control  $p$  is related to  $u$  through  $p = \frac{2-u}{2W^2} \cdot 1$ .

#### B. Adaptive controller design using output feedback

Since the Internet is a large scale complex system [14], the state variable window size can't be measured locally. It is more suitable to deploy a controller which only needs the measurement of queue length. We therefore propose the following output feedback controller, where  $q$  is considered as the measured output, and the window size  $W$  is an unmeasured state variable.

*Assumption 1:* In the plant dynamics (1) and (2), window size  $W$  works in the vicinity of its equilibrium.

The above system (1) and (2) is approximated by the linearized system of (1) and

$$\dot{W} = \frac{1}{\tau} - \frac{W^{*2} + 2}{2\tau} p$$

Then we introduce the transformation:

$$\begin{aligned} \eta_1 &= q \\ \eta_2 &= \frac{N}{\tau}(W - W^*) \end{aligned}$$

Thus the plant dynamics is transformed into:

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= \frac{N}{\tau^2} - \frac{N(W^{*2})}{2\tau^2} p \end{aligned}$$

Define:

$$\xi = \eta_2 - K_1,$$

where  $K_0$  is the tuning parameter for filter bandwidth.

$$\begin{aligned} \dot{\xi} &= \dot{\eta}_2 - K_1 \dot{\eta}_1 = \frac{N}{\tau^2} - \frac{N(W^{*2})}{2\tau^2} p - K_2 \xi \\ &= -K_0 - K_2 \eta_1 + \frac{N}{\tau^2} - \frac{N(W^{*2})}{2\tau^2} p \end{aligned} \quad (5)$$

Then introduce the filters:

$$\dot{\lambda}_1 = -K_1 \lambda_1 + 1 \quad (6)$$

$$\dot{\lambda}_2 = -K_2 \lambda_2 + p \quad (7)$$

$$\dot{\lambda}_3 = -K_3 \lambda_3 + K_1 \quad (8)$$

Consider

$$\begin{aligned} e &= \xi - \frac{N}{\tau^2} \lambda_1 + \frac{N(W^{*2})}{2\tau^2} \lambda_2 + K_3 \lambda_3 \\ \dot{e} &= \dot{\xi} - \frac{N}{\tau^2} \dot{\lambda}_1 + \frac{N(W^{*2})}{2\tau^2} \dot{\lambda}_2 + K_3 \dot{\lambda}_3 \\ &= -K_0 \left( \xi - \frac{N}{\tau^2} \lambda_1 + \frac{N(W^{*2})}{2\tau^2} \lambda_2 + K_3 \lambda_3 \right) \\ &= -K_0 e \end{aligned} \quad (9)$$

The augmented system for output feedback design is:

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 = \xi + K_1 \\ &= -\frac{N(W^{*2})}{2\tau^2} \lambda_2 + \frac{N}{\tau^2} \lambda_1 - K_3 \lambda_3 + e + K_1 \\ \dot{\lambda}_2 &= -K_2 \lambda_2 + p \end{aligned} \quad (10)$$

From here, we can invoke recent advances in adaptive non-linear control [12] and [13] to complete our feedback design. For notational simplicity, set

$$\begin{cases} \theta_1 = \frac{2\tau^2}{N(2W^{*2}) + 1} \\ \theta_2 = \frac{2}{2W^{*2} + 1} \\ \theta_3 = \frac{N(2W^{*2})}{2\tau^2} + \frac{1}{\theta_1} \\ \theta_4 = \frac{N}{\tau^2} = \frac{\theta_2}{\theta_1} \end{cases}$$

as unknown parameters in our model. Then for any given desired queue length  $q^*$ , apply the following coordinates change to the system (10).

$$\begin{cases} x_1 = \eta_1 - q^* \\ x_2 = -\lambda_2 + K_1 \hat{\theta}_1 (\eta_1 - \lambda_3) - \hat{\theta}_2 \lambda_1 + c_1 x_1 \end{cases}$$

where  $\hat{\theta}_1$  and  $\hat{\theta}_2$  are estimates of  $\theta_1$  and  $\theta_2$  whose dynamics will be determined later.  $c_1$  is a control gain tuning parameter.

The new dynamics are established as

$$\begin{cases} \dot{x}_1 = \dot{\eta}_1 = e - \frac{c_1}{\hat{\theta}_1} x_1 \\ \quad + \frac{1}{\hat{\theta}_1} [x_2 + K_1(\theta_1 - \hat{\theta}_1) \eta_1 - \lambda_3] - (\theta_2 - \hat{\theta}_2) \lambda_1 \\ \dot{x}_2 = K_2 - p + K_1 \hat{\theta}_1 (\eta_1 - \lambda_3) \\ \quad + (\hat{\theta}_1 K_1 + c_1) \frac{1}{\hat{\theta}_1} \lambda_2 + K_1 (\eta_1 - \lambda_3) - \frac{N}{\tau^2} \lambda_1 + e \\ \quad + K_2 \hat{\theta}_1 (\lambda_3 - \eta_1) - \hat{\theta}_2 \lambda_1 + \hat{\theta}_2 (-K_1 + 1) \end{cases}$$

Introduce  $u$  as a nominal control and consider

$$p = K_2 + K_1 (\eta_1 - \lambda_3) \hat{\theta}_1 + c_1 \hat{\theta}_2 \lambda_1 + \hat{\theta}_2 (-K_1 + 1) - u$$

Thus the next step is to design a control  $u$  to adaptively stabilize

$$\begin{cases} \dot{x}_1 = e - \frac{c_1}{\hat{\theta}_1} x_1 \\ \quad + \frac{1}{\hat{\theta}_1} [x_2 + K_1(\theta_1 - \hat{\theta}_1) \eta_1 - \lambda_3] - (\theta_2 - \hat{\theta}_2) \lambda_1 \\ \dot{x}_2 = (\hat{\theta}_1 K_1 + c_1) e + (\hat{\theta}_1 K_1 + c_1) - \theta_3 \lambda_2 + \theta_4 \lambda_1 - u \end{cases} \quad (11)$$

Now choose  $u$  and apply parameters estimating laws as follows.  $\hat{\theta}_1$  and  $\hat{\theta}_2$  are as previously defined.  $\hat{\theta}_3$  and  $\hat{\theta}_4$  are estimates for  $\theta_3$  and  $\theta_4$  respectively.

$$u = -[c_2 + (\hat{\theta}_1 K_1 + c_1)^2] x_2 + (\hat{\theta}_1 K_1 + c_1) \hat{\theta}_3 \lambda_2 - \hat{\theta}_4 \lambda_1 - x_1$$

$$\begin{cases} \dot{\hat{\theta}}_1 = \gamma_1 K_1 (\eta_1 - \lambda_3) x_1 \\ \dot{\hat{\theta}}_2 = \gamma_2 \lambda_1 x_1 \\ \dot{\hat{\theta}}_3 = -\gamma_3 (\hat{\theta}_1 K_1 + c_1) \lambda_2 x_2 \\ \dot{\hat{\theta}}_4 = \gamma_4 (\hat{\theta}_1 K_1 + c_1) \lambda_1 x_2 \end{cases} \quad (12)$$

$c_2 > 0$  is a control gain and  $\gamma_1, \gamma_2, \gamma_3, \gamma_4 > 0$  are adaptation tuning parameters.

Consider the Lyapunov function candidate

$$V = \frac{\theta_1}{2} x_1^2 + \frac{x_2^2}{2} + \sum_{i=1}^4 \frac{(\hat{\theta}_i - \theta_i)^2}{2r_i} + \frac{\rho}{2} e^2 \quad (13)$$

$$c_1 > 1, c_2 > 0$$

$$r_i > 0 \text{ where } i = 1, 2, 3, 4$$

$$\rho = \frac{1}{4K_1} +$$

After some algebra, it can be verified through (11), (12), (13) and (9) that

$$\begin{aligned} \dot{V} &= -c_1 x_1^2 - c_2 x_2^2 + K_1 (\theta_1 - \hat{\theta}_1) \eta_1 - \lambda_3 x_1 \\ &\quad + (\theta_2 - \hat{\theta}_2) \lambda_1 x_1 + (\hat{\theta}_1 K_1 + c_1) \lambda_2 x_2 (\hat{\theta}_3 - \theta_3) \\ &\quad + (\hat{\theta}_1 K_1 + c_1) \lambda_1 x_2 (\theta_4 - \hat{\theta}_4) - \sum_{i=1}^4 \frac{(\hat{\theta}_i - \theta_i) \dot{\hat{\theta}}_i}{r_i} \\ &\quad - (\hat{\theta}_1 K_1 + c_1)^2 x_2^2 + \theta_1 x_1 e + (\hat{\theta}_1 K_1 + c_1) x_2 e - \rho e^2 \end{aligned} \quad (15)$$

By completing the squares

$$\begin{cases} \theta_1 x_1 e \leq x_1^2 + \frac{\theta_1^2 e^2}{4} \\ x_2 (\hat{\theta}_1 K + c_1) e \leq x_2^2 (\hat{\theta}_1 K + c_1)^2 + \frac{e^2}{4} \end{cases} \quad (16)$$

Combing in (14), (15) and (16), it is clear that

$$\dot{V} \leq -(c_1 - 1)x_1^2 - c_2 x_2^2 - \left(\rho - \frac{1 + \theta_1^2}{4}\right) e^2 \leq 0$$

Thus the control  $p$  will adaptively asymptotically stabilize  $x_1$  and  $x_2$  at the origin with unknown network parameters. The original system therefore achieves asymptotic convergence of window size and queue length, where equilibrium queue length is a design freedom.

*Remark 1:* In the filter dynamics (6-8), (8) is the same output filter used by the RED controller where  $K = 0$  is the filter constant. The control input of the RED controller is of the form  $L_{re} \cdot \lambda_3$ , where  $L_{re}$  is a RED design parameter,  $\lambda_3$  is the filter output. According to [15], [6], the equilibrium queue length of the closed loop system-TCP/RED is determined by network parameters and the RED parameter  $L_{re}$ . One benefit of our controller over RED is that the equilibrium queue length of our control system can be chosen freely. On the other hand, the TCP/RED control system achieved linear stability [6]. We hope the new control law we design using Lyapunov design method can provide a larger domain of attraction than RED. The other two filters (6,7) are introduced to handle the unknown parameters in (5).

Simulation results in Figure 1 show that source window size  $W$ , queue length  $q$  and loss probability  $p$  converge to the unique equilibrium values. The parameter setting are:

$$\begin{aligned} N &= \theta, \quad \tau = 0.1 \text{ sec}, \quad C = \theta \quad \text{packets/sec} \\ B &= \emptyset \quad \text{packets, packets length} = \emptyset \quad \text{bytes,} \\ \text{segment size} &= \emptyset \quad \text{bytes} \end{aligned}$$

The equilibrium window size and packets dropping probability are  $W^* = 10$  segments and  $p^* = 0.02$  in the above setting. After 3 seconds and 6 seconds, the number of sources changes from 50 to 80, then *round trip time* change from 100 ms to 150 ms,  $W^*$  and  $p^*$  change correspondingly. Throughout the simulation, we chose the operating value for buffer queue length to be  $B = 2$ . The purpose of this intuitive choice is to achieve both sufficient link capacity utilization and to avoid buffer congestion. The system trajectory converges to the desired operating point. 20% background traffic is present. Thus the changes of network parameters do not affect stability though operating point conditions changed.

#### IV. COMPARISON WITH EXISTING AQM SCHEMES

To compare our adaptive output feedback controller with existing AQM schemes, we give the simulation results for systems using Drop-tail and RED controllers with the same parameter setting. The models for Drop-tail and RED are taken from [14].

The simulation in Figure 2 shows that window size and buffer queue is stabilized while 20% of non-TCP sources are

present. However, the equilibrium queue length is close to full buffer size as revealed by simulations in Figure 2 and 3. It may cause packet loss in face of traffic bursts.

For RED, the simulation in Figure 4 and 5 showed a limit cycle, as compared to the asymptotic regulation result of our adaptive output feedback system in Figure 1. For more theoretic discussions of the destabilizing effects of RED, please refer to Ranjan, Abed and La [16].

#### V. CONCLUSION

With the help of Lyapunov design method, adaptive feedback linearization and filter techniques, nonlinear controllers are designed and are shown to achieve asymptotic stability and enjoy better performance than both drop-tail and RED. The window size and queue length are shown to converge despite the uncertainty of network parameters. Nevertheless, more research is needed to take into account uncertainties of other kinds such as un-modelled dynamics. In the proposed adaptive design, the estimator tuning parameter  $\gamma_i$ ,  $i = 1, 2, 3, 4$  and the controller gains  $c_1$  and  $c_2$  should not be chosen to be extremely large so as to avoid exciting the un-modelled dynamics and therefore prevent instability. In addition, adaptive techniques like dead-zone, sigma-modification and projection should be used in practice, to avoid any possible parameter drift instability. Note that we have only discussed a single bottleneck link network with homogeneous sources. We omitted control input saturation in our design. In the future, we will address the control input saturation constraint, multiple bottleneck link networks and delay.

It is our belief that the adaptive feedback approach and nonlinear analysis advocated here should provide benefits to future networking technical developments and serve as guidance for decentralized protocol designs.

#### REFERENCES

- [1] T. Alpcan and T. Basar, "Global Stability Analysis of End-to-End Congestion Control Schemes for General Topology Networks with Delay," in Proceedings of the 42nd IEEE Conference on Decision and Control, pp. 1092-1097 Maui, Hawaii, December 2003.
- [2] Y. Chait, C.V. Hollot, V. Misra, S. Oldak, D. Towsley and W.Gong, "Fixed and adaptive model-based controllers for active queue management", in Proceedings of American Control Conference, vol. 4, pp. 2981-2986, June 2001.
- [3] S. Floyd and R. Gummadi and S. Shenker, "Adaptive RED: An Algorithm for Increasing the Robustness of RED", Technical Report, 2001
- [4] X. Fan, M. Arcak and J. Wen, "Lp-stability and delay robustness of network flow control," In Proceedings of the 42nd IEEE Conference on Decision and Control, pp. 3683-3688, Maui, Hawaii, December 2003.
- [5] Yuan Gao, Jennifer C. Hou, "A State Feedback Control Approach to Stabilizing Queues for ECN-Enabled TCP", INFOCOM 2003, San Francisco, California, April 2003.
- [6] C.V. Hollot, V. Misra, D. Towsley and W. B. Gong, "A Control Theoretic Analysis of RED", in Proceedings of INFOCOM, pp.1510-1519, April 2001.
- [7] C.V. Hollot, V. Misra, D. Towseley, W. B. Gong, "On Designing Improved Controllers for AQM Routers Supporting TCP Flows", in Proceedings of IEEE Infocom, 2001.
- [8] C.V. Hollot, Y. Chait, "Nonlinear stability analysis for a class of TCP/AQM network", in Proceedings of the 40th IEEE Conference on Decision and Control, vol. 3, pp. 2309-2314, 2001.
- [9] V. Jacobson, "Congestion Avoidance and Control", in Proceedings of SIGCOMM '88, Palo Alto, CA, Aug. 1988.

- [10] H. K. Khalil, *Nonlinear System*, 3rd edition, Prentice Hall, Upper Saddle River, 2002.
- [11] F. P. Kelly, "Mathematical modeling of the Internet", in Proceedings of the 4th International Congress on Industrial and Applied Mathematics, 1999.
- [12] P. V. Kokotovic, "The joy of feedback: Nonlinear and adaptive", *IEEE Control System Magazine*, pp.7-17, June, 1992.
- [13] P. V. Kokotovic, M. Arcaç, "Constructive nonlinear control: a historical perspective", *Automatica*, vol.37, pp. 637-662, 2001
- [14] S. H. Low, F. Paganini and J. C. Doyle, "Internet Congestion Control", *IEEE Control Systems Magazine*, pp.28-43, Feb. 2002
- [15] V. Misra, W. Gong, D. Towsley, "A Fluid-based Analysis of a Network of AQM Routers Supporting TCP Flows with an Application to RED", in Proceedings of ACM SIGCOMM, September, 2000.
- [16] P. Ranjan, E.H. Abed, and R.J. La, "Nonlinear instabilities in TCP-RED", in Proceedings of INFOCOM, vol. 1, pp. 249 -258, 2002.
- [17] J.T. Wen, M. Arcaç, "A unifying passivity framework for network flow control", in Proceedings of INFOCOM 2003, vol. 2, pp. 1156 -1166, April 2003.
- [18] H. Zhang, C.V.Hollot, D. Towsley, V. Misra, "A Self-Tuning Structure for Adaptation in TCP AQM Networks", in Proceedings of the International Conference on Measurements and Modeling of Computer Systems, pp.302-303, San Diego, CA, USA, June 9-14, 2003.

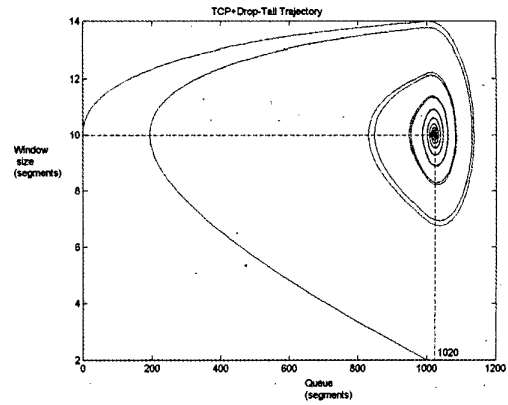


Fig. 3. TCP/Drop-tail system trajectories

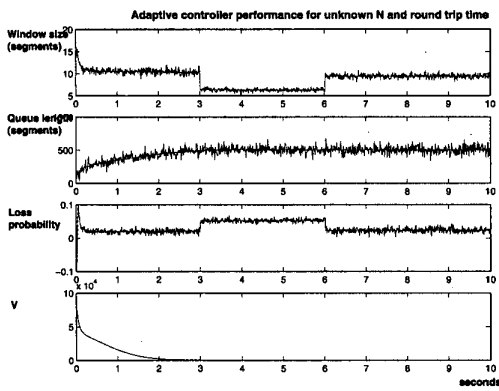


Fig. 1. Adaptive system response

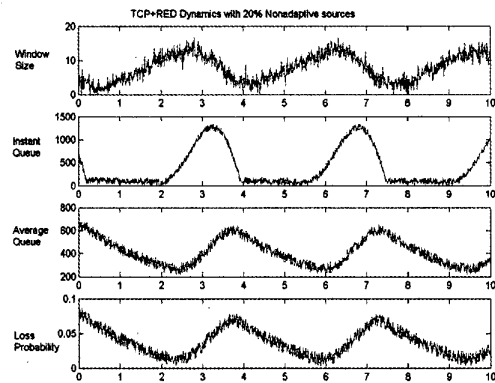


Fig. 4. TCP/RED system response

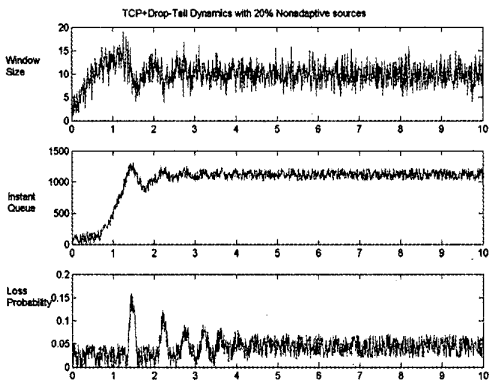


Fig. 2. TCP/drop-tail system response

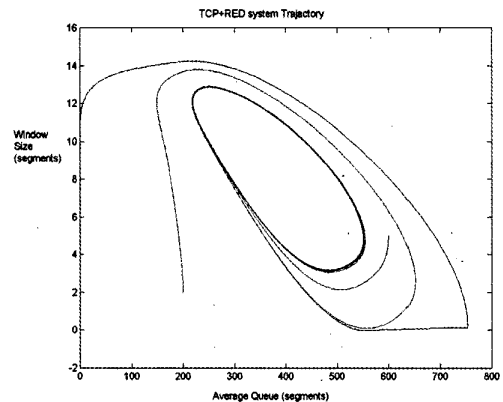


Fig. 5. TCP/RED system trajectories