A COLLISION RESOLUTION ALGORITHM FOR RANDOM ACCESS CHANNELS USING MULTIPLE TRANSMISSION LEVELS

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ABSTRACT

Networks with random access protocols offer a fair access to all users, they also show a robustness towards node failures. These features make them an attractive solution for computer applications. A vast amount of present day networks work under this scheme, being Ethernet one of the most significant technological contributions in this area. Collision Resolution Protocols are a special kind of random access schemes, offering the advantage that stability of the network is guaranteed, provided that the average packet input rate does not exceed a given limit.

Throughput is defined as the number of successful transmissions in a given time period. Pippenger[1] *offered a non-constructive proof that a throughput of 1 is achievable in the limit if users are able to detect the number of transmissions involved in a collision. A constructive method to achieve a throughput of .532 was found by Georgiadis and Papantoni-Kazakos*[2]*, a result which was improved to .553 by Kessler and Sidi*[3] *adding information to each of the transmitted packets. Panwar developed two constructive methods of achieving a throughput of 1 (in the limit) by means of multiple transmission powers*[4] *and the use of detecting matrices*[5]*.*

The purpose of this paper is to present a protocol that offers an interesting tradeoff when limited transmission power levels are permitted. Using this protocol, the designer is able to raise the achievable throughput to .553 by using 2 different transmission powers and to .578 choosing one of at most 3 selectable transmission powers. We also outline the method to achieve higher throughputs increasing the complexity of the transmitters. The scheme is applicable in today's networks based on fiber optics and spread spectrum systems, where the number of simultaneous transmissions can be detected.

INTRODUCTION

Consider a large number of users accessing at random a single time-slotted channel to transmit fixed-length messages, called packets. We suppose that the total traffic generated by these users can be modelled as a Poisson process with average arrival rate, λ. Assume that single transmissions are always successful. However, if 2 or more messages are transmitted simultaneously, they will collide and are thereby rendered unintelligible. Each user station monitors the channel activity and when it realizes that its transmission has collided, retransmits the message in accordance with the rules of the protocol. *Collision Resolution Protocols (CRP)* are algorithms that arbitrate transmissions and retransmissions of packets, so that eventually all packets involved in a collision are guaranteed successful transmission and all users are aware of this fact. Users who experienced new packet arrivals while a collision resolution process is running are inhibited to transmit and may transmit their packets only once there are no retransmissions pending.

Throughput is defined as the number of successful transmissions in a given time period. CRP's are algorithms that guarantee stable performance as long as the arrival rate is kept below the expected achievable throughput.

Capetanakis[6], Hayes[7] and Tsybakov and Mikhailov[8] proposed almost concurrently a Binary-Tree-CRP. It resolved collisions having users flip a binary coin, granting retransmission rights to those flipping a zero. Users flipping a one had to wait for retransmission until those flipping a zero had sucessfully transmitted their packets. This algorithm was shown to be stable, provided the total packet arrival rate was kept below 0.346. This was a major breakthrough for random access protocols, which up to that point could reach channel saturation with probability 1.

Gallager[9] (and independently Tsybakov and Mikhailov[10]) improved the tree algorithm proposed by Capetanakis. Instead of using a splitting algorithm based on the outcome of flipping coins, they proposed that each time a collision is detected to split the arrival interval into two. Only those packets arriving in the first sub-interval were enabled for retransmission. This ensured that packets were always transmitted in the order of their arrival, a property that inspired the authors to name this algorithm a *First-Come-First-Serve (FCFS)* protocol. The expected throughput for this algorithm is 0.4871[11].

Georgiadis and Papantoni-Kazakos[2] could improve the performance of the FCFS CRP by assuming that the number of colliding packets could be established at each contention slot. This information was obtained by energy level detectors at the receiver and was used to optimize the splitting based on the number of contending transmissions. The protocol was accordingly named *Collision Resolution with Additional Information (CRAI)*. Georgiadis was able to prove that a maximum throughput of 0.53237 was achievable using this strategy.

However, Pippenger[1] published a non-constructive proof that a throughput of 1 was achievable if the collision multiplicity is known. Thereafter Yates[12] proved that this throughput was possible at the expense of infinitely long delays.

A COLLISION RESOLUTION ALGORITHM USING MULTIPLE TRANSMISSION LEVELS (CRA/MTP)

We are presenting a CRA with collision multiplicity detection and, additionally, multiple transmission powers at the transmitter. We show that adding complexity to the transmitter it is possible to improve throughput without increasing delays. Panwar[4] showed that this tradeoff, in the limit, achieves a throughput of one, although his scheme is not as efficient as the one presented here in assigning transmission power.

To illustrate how this algorithm resolves collisions we will make use of two time axis: on one time axis packet arrivals are recorded, whereas channel activity is monitored on the other one. Packets arriving while a collision resolution process is in progress are delayed for transmission until all packets involved in that collision have been transmitted successfully. Users that experienced packet arrivals in the next available time interval transmit in the immediate time slot. Let N be the number of users that transmit a packet. If $N = \{0,1\}$ then either a slot is wasted, or a successful transmission takes place, respectively. If $N \geq 2$, transmissions collide and cannot be received correctly, so that these packets will have to be retransmitted. Each of the involved users flips a fair G(N)-sided die which distributes them into G(N) separate groups. An equivalent assignment is to divide the enabled arrival time interval into G(N) subintervals of equal size[11]. A unique transmission power level is associated to each group. When users retransmit their packets in the next available transmission slot, the exact membership of a group is extracted from the total energy level. This procedure of splitting, associating transmission levels to each resulting group, transmitting and establishing individual group membership, is repeated until all packets experience single transmissions.

Georgiadis' scheme[2] can be analized as one for which $G(N) = 2$ and the corresponding throughput is 0.53237. In this case a single power level is required: members of one group do not transmit while the members of the other group do.

Larger values of G(N) require more than one transmission level and convey more information, since users involved in a collision are split into groups of smaller size, thus reducing the probability of future collisions. These transmission levels must satisfy the condition that composition of each group can be established from the received energy level. As mentioned before, Panwar[4] proposed a scheme in which group members can be identified by selecting transmisision powers taken from the geometric series of powers of 2. By carrying the enabled arrival time division process to the limit it was shown that a throughput approaching 1 was possible. In this paper we look for a more efficient way to achieve the desired performance.

Consider the mathematical structure of non-negative integers known as B_h sequences. B_h sequences are interesting in this case because they give a unique sum for any choice of h elements from the sequence, allowing repetitions. Better known B_h sequences are B_2 sequences[14-16]. We are interested on B_h sequences with $h \geq 3$ to be able to split users into more than 2 groups. For instance, a B_3 sequence of four elements is

$$\{0,1,7,11\}$$

Adding any 3 elements of this sequence, allowing repetitions, will generate a sum which is unique to the chosen elements. The requirement $G(N) > 2$ can be met choosing transmission power levels from a suitable B_h sequence.

To illustrate this idea consider that 4 packet transmissions collided in an attempt to access the channel. A possible situation is depicted in Figure 1, where the arrival time axis has been depicted horizontally, while snapshots of the slotted transmission time axis are shown in downward progression. We will describe the collision resolution process by analyzing Figure 1 according to the transmission time snapshots. We start at slot k, where a new time interval is enabled on the arrival time axis so that users that experienced arrivals in that interval may transmit. All 4 users of this example (labeled A, B, C, and D) transmit and collide in transmission slot k. Everybody involved now flips a G(N)-

sided dice. As it will be shown later, according to the CRA/MTP protocol, it will be a 3-sided dice. The equivalent procedure[11] of splitting the original interval in 3 sub-intervals of equal size is shown in Figure 1. Also, as it will be shown later, the transmission levels are chosen from the B_4 sequence { 0, 1, 5}. User A does not transmit, since he belongs to the first group, B transmits with a power level of 1 and C, D transmit with power level 5 in transmission slot k+1. The receiver detects an energy level of 11, and thus it is known that the first and the second intervals experienced a single arrival and that the 2 remaining ones are in the third interval. Group 1 is enabled first and A transmits successfully in trans-mission slot k+2. Then B transmits successfully (trans-mission slot k+3). Since Group 3 has 2 arrivals, it is split again before its members transmit. When the group is split, C and D belong to different groups and therefore transmit success-fully in slots k+4 and k+5, respectively. Since it has taken 6 transmission slots to resolve the contention of 4 arrivals, the throughput in this case is 4/6 = 0.667.

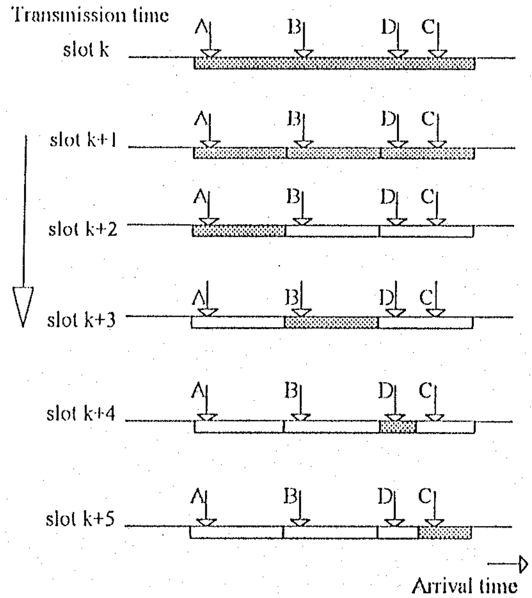

Figure 1. Collision resolution using energy detection at the receiver and 2 selectable transmission powers.

For this scheme to work efficiently from a practical point of view, it is important to reduce the maximum trans-mission power. Since this power level is determined by the largest value of a given B_h sequence ($h > 2$), research was conducted to find those sequences with its largest value as small as possible. We have not been able to see this information published and therefore summarized our results in Appendix 1.

Define an epoch as the number of transmission slots needed to resolve a collision of N transmissions. The length of an epoch will be dependent on the choice of **G(N)**, which in turn determines the B_h sequences to be used, thus fixing the required transmission powers. We decided to investigate throughput performance under the following assump-tions: infinite many energy levels can be detected at the receiver and a certain power limitation at the transmitter.

Define $L_p(N)$ as the expected length of an epoch when it has been detected that N transmissions collided. The subscript p stands for the highest permissible transmission power level and emphasizes the fact that the algorithm is sensitive to this value. Define $L_p(0) = 0$. To take into account the extra slot that is required to transmit a packet that is the only member of a group from a split, let $L_p(1) = 1$.

In order to illustrate our analysis, values of $L_p(N)$ for the case of transmission power level restricted to 6 times the nominal level will be developed first. Define

$$Q_i(N) = \binom{N}{i} \sigma_N^i (1 - \sigma_N)^{N-i} \tag{1}$$

where σ_N is a parameter to be optimized whenever G(N) = 2. When 2 transmissions collide, the enabled arrival interval has to be split in 2. The arrival times will distribute according to a binomial distribution, in a pattern that can either be ⟨2, 0⟩, ⟨1, 1⟩ or ⟨2, 0⟩ , where the numbers in brackets indicate the amount of arrivals in the first and second subinterval, respectively. Solving for this case[2], the minimum average number of slots required to successfully transmit them is given by equation (2).

$$\begin{gathered} L_6(0) = 0 \\ L_6(1) = 1 \\ L_6(2) = 1 + Q_0(2)L_6(2) + 2Q_1(2)L_6(1) + Q_2(2)L_6(2) \end{gathered} \tag{2}$$

The right hand side of equation (2) contains $L_6(2)$. This term can be carried over to the left hand side, yielding equation (3).

$$L_6(2) = \frac{1 + 2Q_1(2)}{1 - Q_0(2) - Q_2(2)} \tag{3}$$

The minimum value of $L_6(2) = 3$ and it is achieved for $\sigma_2 = ½$. Applying this technique to find the value of $L_6(3)$, yields $L_6(3) = 4.78795$ for a value of $\sigma_3 = .41188$, as was derived by Georgiadis[2].

There is an advantage in this case if the contention interval is split in 3 when 4 or 5 packet transmissions collide. If 4 4 packets are transmitted simultaneously, the B_4 sequence picked is {0, 1, 5}. Using the same notation as for the $L_6(2)$ case, arrivals will distribute in one of the following ways ⟨4, 0, 0⟩, ⟨3, 1, 0⟩, ⟨3, 0, 1⟩, ⟨2, 2, 0⟩,......⟨0, 1, 3⟩ or ⟨0, 0, 4⟩. Since the probability that one of this events may happen follows the multinomial distribution, $L_6(4)$ is given by the equation (4).

$$L_6(4) = \frac{\left[24L_6(3) + 72L_6(2) + 87 + 3^4\right]}{3^4 - 3} = 6.3963 \tag{4}$$

This expression was derived considering that a single successful transmission is achieved if arrivals distribute according to ⟨3, 1, 0⟩ or ⟨3, 0, 1⟩, since users belonging to the first group do not transmit..

For all values $N \geq 6$, G(N) = 2 and thus $L_6(N)$ for all $N \geq 6$ is found by performing a binary weighted split as in Georgiadis CRAI[2].

With this example in mind, more general situations can be considered and the corresponding equations will be developed. These equations depend on the value of G(N). For instance, if G(N) = 2, it is possible to use the expression derived by Georgiadis[2], which has been included, adapting it's notation to make it consistent with ours.

$$L_p(N)_{G(N)=2} = \begin{cases} N & \forall 0 \leq N \leq 2 \\ \dfrac{1 - Q_1(N) + \sum_{i=0}^{N-1}\left[Q_i(N) + Q_{N-i}(N)\right]L_p(i)}{1 - Q_0(N) - Q_N(N)} & otherwise \end{cases} \tag{5}$$

As before, the parameter σ_N of this equation is optimized in each case to yield the smallest $L_p(N)$.

If users are split into $G(N) > 2$ groups, one has to keep in mind that arrival instants scatter according to a multinomial distribution. Thus following a similar procedure as the one used to get equation (4), the expected length of an epoch can be obtained, given that $G(N) > 2$. Equation (6) summarizes this result.

$$L_p(N)_{G(N)>2} = \frac{G(N)^N + 2NL_p(N-1) + \sum\limits_{\substack{i_1+i_2+..i_{G(N)}=N \\ i_1 \neq N,...i_{G(N)} \neq N}} \binom{N}{i_1,..i_{G(N)}} (L_p(i_1)+..+L_p(i_{G(N)}))}{G(N)^N - G(N)} \qquad (6)$$

We now consider the effect of limiting the transmission power levels to a maximum of 6, 8, 10 and 12 on the length of an epoch. These choices, although arbitrary, show clearly how throughput performance depends on the admissible power levels. The results derived from this analysis are summarized in Table 1.

TABLE 1. Length of epochs as a function of maximum allowable power levels.

Maximum power level	N = Collision multiplicity	G(N)	B_h sequence	$L_p(N)$ Length of Epoch
6	4	3	{0,1,6}	6.396292
	5	3	{0,1,6}	8.164345
8	4	3	{0,1,8}	6.396292
	5	3	{0,1,8}	8.164345
	6	3	{0,1,8}	9.912486
	7	3	{0,1,8}	11.65529
10	4	3	{0,1,10}	6.396292
	5	3	{0,1,10}	8.164345
	6	3	{0,1,10}	9.912486
	7	3	{0,1,10}	11.65529
	8	3	{0,1,10}	13.38815
	9	3	{0,1,10}	15.12953
12	3	4	{0,1,7,11}	4.516667
	4	3	{0,1,12}	6.325641
	5	3	{0,1,12}	8.019872
	6	3	{0,1,12}	9.708445
	7	3	{0,1,12}	11.39895
	8	3	{0,1,12}	13.08763
	9	3	{0,1,12}	14.79249
	10	3	{0,1,12}	16.77439
	11	3	{0,1,12}	17.02144

The values of $L_p(N)$ shown on the right column in Table 1 have been derived using equation (6) and $L_p(0) = 0$, $L_p(1) = 1$, $L_p(2) = 3$ and $Lp(3) = 4.78795$. When transmission power is limited to a maximum of 12 (last group in Table 1), it is possible to split the enabled interval into 4 groups when 3 packet transmissions collide. In that case a B_4 sequence is used and equation (6) is applied to find $L_{12}(3)$. This yields a value of $L_{12}(3) = 4.516667$, which is less than $Lp(3) = 4.78795$, which is obtained with a split in 2 sub-intervals.

All values of $L_p(N)$ with N greater than the entry on the second column of Table 1 are derived using equation (5), since a split of 2 is best in these cases, given the power constraint. Tables of $L_p(N)$ for values of N ≤ 80 have been published by Grote[17].

Rom and Sidi[19] developed a closed form expression for $L_p(N)$ for the special case of G(N) = 2 . However, no advantage is gained by using this approach in the context of this analysis, which is why it has not been included.

To determine the maximum throughput for the general case (0 ≤ N < ∞), the expectation of $L_p(N)$ for any value of N has to be found. The technique used to analyze these cases was introduced by Massey[18]. For completeness we will summarize that procedure as we go along to establish throughput performance of this CRA/MTP protocol.

The expressions derived so far don't make it easy to establish the throughput for all N. However, the results obtained from the computation of $L_p(N)$ for values of N ≤ 80 show that a linear approximation of $L_p(N)$ as a function of N seems possible[17]. To take into account the slight departure that $L_p(N)$ experiences from a linear approximation, linear upper and lower bounds for $L_p(N)$ are used. These bounds take the form of equation (7).

$$\alpha_{M,b,p} \cdot N - 1 \le L_p(N) \le \alpha_{M,a,p} \cdot N - 1 \quad \forall N \ge M \tag{7}$$

M defines a starting value from which point on we may consider equation (7) to be applicable. The choice of M is critical, it is best if it is picked so that equation (7) can be replaced into equation (5). The two parameters $\alpha_{M,a,p}$ and $\alpha_{M,b,p}$ define the slope of the upper and lower bound of $L_p(N)$, respectively. The -1 constant in equation (7) stems from the fact that for large N, the best split in two groups is one that leaves N/2 users in each subgroup.

Additional advantage of picking larger values for M in equation (7) is that the bounds get tighter. Since $\alpha_{M,a,p}$ and $\alpha_{M,b,p}$ are found in identical ways, only the procedure to find $\alpha_{M;b,p}$ will be derived. Basically, this is done by replacing the lower bound expression of (7) into (5) for all N ≥ M and making sure that the left-hand side of equation (7) is fulfilled..

Substitution of equation (7) into (5) for $N \ge M$ yields equation (8), after some work.

$$L_p(N) \ge \alpha_{M,b,p} \cdot N - 1 + \frac{\sum_{i=0}^{M-1} [Q_i(N) + Q_{N-i}(N)][L_p(i) - \alpha_{M,b,p} \cdot i + 1] - Q_1(N)}{1 - Q_0(N) - Q_N(N)} \tag{8}$$

Comparing this expression with equation (7), one concludes that the numerator of the fraction must be 0. Thus we get equation (9).

$$\alpha_{M,b,p} = \inf_{N \ge M} \min_{\sigma} \left[\frac{\sum_{i=0}^{M-1} [Q_i(N) + Q_{N-i}(N)][L_p(i) + 1] - Q_1(N)}{\sum_{i=0}^{M-1} [Q_i(N) + Q_{N-i}(N)] \cdot i} \right] \tag{9}$$

Similarly equation (10) can be derived.

$$\alpha_{M,a,p} = \sup_{N \ge M} \min_{\sigma} \left[\frac{\sum_{i=0}^{M-1} [Q_i(N) + Q_{N-i}(N)][L_p(i)+1] - Q_1(N)}{\sum_{i=0}^{M-1} [Q_i(N) + Q_{N-i}(N)] \cdot i} \right] \tag{10}$$

Choosing $M = 30$ produces sufficiently tight upper and lower bounds for $L_p(N)$[17]. These bounds have been summarized in Table 2.

TABLE 2. Values for upper and lower bounds of $L_p(N)$

Maximum Power Level	p = 6	p = 8	p = 10	p = 12
$\alpha_{30,b,p}$	1.848983	1.825196	1.807988	1.726033
$\alpha_{30,a,p}$	1.849002	1.825322	1.808257	1.730218

Equation (11) shows how it is possible to rewrite equation (7) so that all values of $L_p(N)$ can be included[2] using the parameters $\alpha_{M,a,p}$ and $\alpha_{,M,b,p}$.

$$\begin{gathered} \alpha_{M,b,p} \cdot N - 1 + \sum_{i=0}^{M-1} c_{ib}\delta_{iN} \le L_p(N) \le \alpha_{M,a,p} \cdot N - 1 + \sum_{i=0}^{M-1} c_{ia}\delta_{iN} \\ where: \quad \delta_{iN} = \begin{cases} 1 & i = N \\ 0 & otherwise \end{cases} \\ c_{ib} = L_p(i) - \alpha_{M,b,p} \cdot i + 1 \quad \forall i \le M-1 \\ c_{ia} = L_p(i) - \alpha_{M,a,p} \cdot i + 1 \quad \forall i \le M-1 \end{gathered} \tag{11}$$

We are now in condition to evaluate the upper and lower bounds on the expected length of an epoch $E[L_p(N)]$. At this point, the initial slot required to establish the multiplicity of the colliding packets will have to be considered. Define $L_{p,t}(N)$ as the expected *total length of an epoch*, including the initial slot that helps to determine the number of colliding packets. Since packets will collide only if 2 or more transmitters attempt transmission, we can write equation (12).

$$\alpha_{M,b,p} \cdot N + \sum_{i=0}^{M-1} c_{ib}\delta_{iN} \le L_{p,t}(N) \le \alpha_{M,a,p} \cdot N + \sum_{i=0}^{M-1} c_{ia}\delta_{iN} \tag{12}$$

The probability of having n arrivals in the enabled interval τ, given that λ is the Poisson arrival rate of packets and defining $x = \lambda\tau$, is shown in equation (13).

$$P_n(x) = e^{-\lambda\tau}\frac{(\lambda\tau)^n}{n!} = e^{-x}\frac{x^n}{n!} \tag{13}$$

Taking the expectation of equation (12) yields equation (14)

$$\alpha_{M,b,p} \cdot x + \sum_{i=0}^{M-1} c_{ib} P_i(x) \le E\left[L_{p,t}(N)\right] \le \alpha_{M,a,p} \cdot x + \sum_{i=0}^{M-1} c_{ia} P_i(x) \qquad (14)$$

For stable performance it is necessary that the expected average length of an epoch $E[L_{p,t}(N)] \le \tau$. But $\tau = x/\lambda$. Using this condition we establish upper and lower bounds on the arrival rate for stable performance. The system will be unstable if equation (15) is satisfied.

$$\lambda \ge \sup_x \left[\frac{x}{\alpha_{M,b,p} x + \sum_{i=0}^{M-1} c_{ib} P_i(x)} \right] = \sup_x \left[\frac{1}{\alpha_{M,b,p} + e^{-x} \sum_{i=0}^{M-1} c_{ib} \frac{x^{k-1}}{k!}} \right] \qquad (15)$$

The rightmost part of equation (15) stresses the fact that for large values of x the upper bound for the maximum expected arrival rate λ is given by $1/\alpha_{M,b,p}$. The expression in the center of equation (15) establishes the condition for the expected arrival rate in terms of the ratio of expected packets arriving in the enabled interval to the expected number of transmission slots needed to transmit them.

The upper bound on an arrival rate that guarantees stable performance is given by equation (16).

$$\lambda \le \sup_x \left[\frac{1}{\alpha_{M,a,p} + e^{-x} \sum_{i=0}^{M-1} c_{ia} \frac{x^{k-1}}{k!}} \right] \qquad (16)$$

Varying **x** it is possible to find the value of optimum throughput performance. Table 3 summarizes throughput performance for the limited power levels that have been considered.

TABLE 3: Throughput performance of the CRA/MTP protocol

Maximum Power Level	1	6	8	10	12
Upper Bound Throughput	.533	.540838	.5478865	.5531011	.5793632
Lower Bound Throughput	.53237	.5408324	.5478485	.5530189	.5783271

The first column of values was published by Georgiadis[2]. Increasing the maximum possible transmission level beyond 12 will further reduce the values of the L_N's for N ≥ 3, achieving higher throughputs. In the limit, as $p \to \infty$, throughput approaches 1, as was shown by Panwar in a less efficient scheme[4].

Figure 2 shows how throughput performance varies according to the selected size of the enabled arrival interval. Notice that x is a measure of the expected number of arrivals being selected, a parameter we have named window size. The larger x is, the longer users have to wait for transmission of their packets. Optimum performance in all cases is reached when $x \rightarrow \infty$. However, Figure 2 shows that near optimum performance is achieved for values that are much less. For instance, a window size of 8 yields near optimum performance for all systems with $p \leq 10$. A window size of magnitude 13 yields near optimum performance for $p = 12$.

The actual size of the enabling interval is $\tau = x/\lambda$.

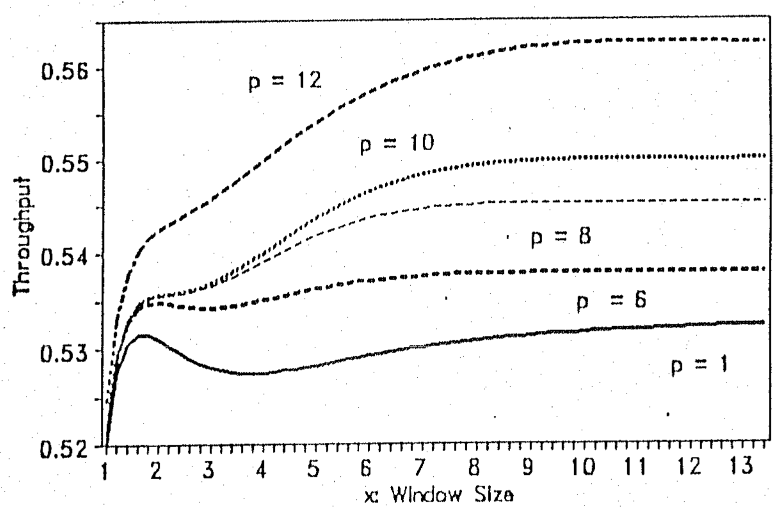

FIGURE 2: Throughput vs. Window Size for CRA/MTP protocol. Notice that throughput improves for large x, in fact maximum throughput is achieved for $x \rightarrow \infty$

SUMMARY AND CONCLUSIONS

We presented a collision resolution protocol which utilizes an infinite number of energy detector levels and limited multiple transmission powers. Depending on the number of colliding packets involved, the algorithm splits the collision resolution interval into two or more groups. Applying the properties of the non-negative integer B_h sequences, the number of users in each group can be found from the total received energy level of the colliding packets. A larger number of sub-intervals increases the efficiency of the protocol since it is possible to create smaller groups, thus accelerating the collision resolution. With power levels restricted to 12, the throughput achieved is larger than those previously published and improvements beyond this point are possible using more transmission power. Finding suitable B_h sequences with elements greater than 12 that will be efficient transmission power remains an open problem.

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APPENDIX 1: TABLE OF DENSE B_h SEQUENCES

We have conducted research on a number of B_h sequences with $h > 2$, with the largest element being as small as possible. The results have been summarized in the following table.

Sequence	Number of elements	Highest sum	A high density sequence
B_3	2	5	{0,1}
	3	12	{0,1,4}
	4	33	{0,1,7,11}
	5	69	{0,1,15,18,23}
	6	135	{0,3,19,34,43,45}
B_4	2	4	{0,1}
	3	20	{0,1,5}
	4	60	{0,1,11,15}
	5	164	{0,1,24,37,41}
B_5	2	5	{0,1}
	3	30	{0,1,6}
	4	110	{0,1,16,22}
	5	360	{0,1,16,66,72}
B_6	2	6	{0,1}
	3	42	{0,1,7}
	4	168	{0,1,22,28}
B_7	2	7	{0,1}
	3	56	{0,1,8}
	4	259	{0,1,29,37}

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La Serena, Chile, Octubre/October 13-16, 1993

Preliminary Program

Partial List of Invited Speakers

- How I Learned to Love the Situation Calculus, Raymond Reiter, Univ. of Toronto, Canada.
- Standards and Technology Integration, Richard Soley, Technical Director of OMG, USA.

Tutorials (in Spanish)

- Object Oriented Databases, José Blakeley, Texas Instruments Labs, USA.
- Deductive Databases, Jorge Lobo, Univ. of Illinois at Chicago, USA.
- Distributed Systems, José M. Piquer, Universidad de Chile.

Research Contributions

Algorithms

- An External Sort Algorithm, Frank Lin, University of Maryland, USA.
- Parallel Update and Search in Skip Lists, Joaquim Gabarro, Conrado Martinez, Xavier Messeguer, Univ. Politecnica de Catalunya, Spain.
- Graph Clustering and Caching, Alberto Mendelzon, Carlos Mendioroz, University of Toronto, Canada.
- Probabilistic Analysis of Addition Subtraction Chains, Raul Gouet, Jorge Olivos, Universidad de Chile.
- Prime Length Symmetric FFTs and Their Computer Implementations, Jaime Seguel, Ricardo Santander, Fredi Palominos, Claudio Fuentealba, Universidad de Santiago, Chile.
- An Algorithm for Finding the Safest Path among Obstacles for Acceleration Constrained Robots, Dajin Wang, Montclair State College, USA.
- A Comparison of Algorithms for the Triangulation Refinement Problem, María-Cecilia Rivara, Patricio Inostroza, Universidad de Chile.
- Use of Genetic Algorithms to Optimize the Cost of Automotive Wire Harnesses, Carlos Zozaya-Gorostiza, Hinurimawan Sudarbo, Luis Fernando Estrada, ITAM, Mexico.
- Applying Genetic Algorithms to the Load-Balancing Problem, Alex Alves Freitas, Junia Coutinho Anacleto, Claudio Kirner, Univ. Fed. de Sao Carlos, Brazil.

- Planning Methodology of Information Systems under Cooperative Design, Antonio Guevara, Universidad de Málaga, Spain.

Software Engineering

- Quality Guided Programming: Integrating Code Reviews with Metrics Analysis of Code, Stefan Biffl, Technical University of Vienna, Austria.
- Integrated-Specifications Analysis, Pablo Straub, Yadran Eterovic, Hugo Espinoza, Cecilia Bastarrica, Univ. Católica de Chile.
- Object Oriented Analysis: A Synthetic Approach, Viviana Rubinstein, Jorge Boria, Liveware, Argentina.
- The Management of a Cooperative Environment, Carlos Aguiar, Ana Carolina Salgado, UFPE, Brazil.
- Combining Instance and Class-Based Descriptions in Hypermedia Authoring, Luis M. Bibbo, A. Díaz, S. Gordillo, Gustavo H. Rossi, LIFIA, UNLP, Argentina.

Petri Nets

- Morphisms to Preserve Structural Properties of Petri Nets, Agathe Merceron, Universidad de Chile.
- Analysis and Modelling of Petri Boxes, Raymond Devillers, Universite Libre de Bruxelles, Belgium.
- Studying the Behaviour of Petri Nets through a Formalization as Term Rewriting Systems, Alberto Paccanaro, Univ. Católica Nuestra Señora de la Asunción, Paraguay.
- Event Modeling with Petri Nets: a Survey and Discussion, Carlos A. Heuser, UFRGS, Brazil.

Knowledge and Logic

- Inheritance and Recognition in the Cummulative Typed System for Knowledge Representation SC, Doris Ferraz de Aragon, Alexandre Evsukoff, M.C. Monard, Inst. de Logica, Filosofia e Teoria da Ciencia - ILTC/UFF, Brazil.
- Metacontrol of the AITEC Traffic Simulator using Situation Semantics, Harold Paredes-Frigolett, AITEC GmbH, Germany.
- Making Argument Systems Computationally Attractive, A.J. Garcia, C.I. Chesnevar, G.R. Simari, Univ. Nacional del Sur, Argentina.
- On Observational Equivalence and Relational Semantics, Fabio da Silva, Universidade Federal de Pernambuco, Brazil.
- Abductive Inference of Plans and Intentions in Information Seeking Dialogues, Paulo Quaresma, Jose Gabriel Lopez, Artificial Intelligence Center, UNINOVA, Portugal.
- Equilibration and Belief Revision: Strategies for Comparative Tutoring and Learning, Flavio M. de Oliveira, Rosa M. Viccari, UFRGS, Brazil.
- Cognitive Maps as Human Computer Interface Design Tools for Learning, Jaime Sanchez, A. Mallegas, Univ. de Antofagasta, Chile.
- Lexical Error Correction using Contextual Linguistic Expectations, Karl Klebetsis, Thomas Grechenig, Technical University of Vienna, Austria.